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Model for Reinvestment Policy in Risk-Free Assets with Various Maturities

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***Abstract*: *Logistic tasks are aimed at the optimal distribution of material, energy, financial and human resources. This research has a narrow field aimed at optimal management of financial resources and their redistribution. Specifically, a reinvestment policy model is derived by maximizing the profit of a business entity. Reinvestment is done with risk-free assets, but they have different maturity periods. This makes it difficult to assess the optimal investment strategy, as reinvestment can be done at the end of the maturity period. This study develops a model for a dynamic control process, which leads to the formalization of a discrete integer time optimization problem. Its solution gives a sequence of investments and a total optimal return. The solution to the problem is illustrated in an EXCEL environment. The added value of this study stems from the formalization and quantification of the model for the reinvestment strategy in the optimization problem.*

***Keywords*: *Investment management; portfolio reinvestment; optimization; decision making.*

1. Introduction

The optimization of investment activities needs to be formalized and must be quantified in the management rules of the investment portfolio. The investment process is easy to approximate as logistics activities, which implies optimal resource allocation. Particularly, the investment process reallocates financial resources by means to achieve a goal, which is quantified as portfolio return. The optimal investment strategy has to be formalized with the definition and solution of an appropriate optimization problem.

The most powerful solutions for an optimal investment strategy are coming from the recommendations of the portfolio theory [1-3]. The portfolio theory reallocates the investment resource between a set of assets by means to maximize the portfolio return and minimize the portfolio risk [4-7].

The portfolio theory is applied not only for the financial domain but also in cases when resource allocation is needed to perform as for product management [8], energy management [9], investments in real estate [10], and project management [11].

The portfolio in content has several assets but their weights in the portfolio are different. Normally, the weight of an asset is evaluated according to its impact on the overall portfolio risk and return. In general, all assets are assumed to mature by the time the investment period is over and portfolio performance has been assessed and evaluated.

A particular investment strategy is the reinvestment case when the income from previous investments with ending maturity yields additional resources [12]. The usage of these additional incomes in a continuous investment policy can benefit the increase of the wealth of the investor.

This study deals with the case of reinvestment in an investment procedure. The purpose of this research is to derive a model for combining several risk-free assets in a predetermined investment horizon. This model should be applied to the definition of an optimization problem that will formalize the optimization reinvestment procedure. The solution to this optimization problem is illustrated, leading to an increase in the investor's wealth through the appropriate use of available resources and reinvestment of the additional income received.

The paper is organized into eight sections. Section 2 assesses the need to formalize the reinvestment management process as a portfolio optimization problem. Section 3 defines the content of the reinvestment portfolio problem. Section 4 derives the reinvestment problem analytically. Section 5 numerically defines and estimates a set of parameters for the reinvestment problem. Section 6 presents the rules for programming the Excel software environment for solving the portfolio reinvestment problem. Section 7 gives the decision results from the reinvestment process when investment asset rates change. Finally, the conclusions in section 8 provide advice for the future development of the derived reinvestment optimization model.

2. Portfolio problem and reinvestment management strategy

Portfolio theory is applied in two important cases: deterministic and stochastic [1]. For the deterministic case, the investment objective is to maximize the portfolio's return. Such a form of portfolio optimization can be used when it is assumed that asset returns are well-defined for the end of the investment horizon. Since this general requirement is not always met, the portfolio return estimate is projected in a region around the mean. The size of this region provides an additional parameter for the asset that determines its risk. Accordingly, such a dimension around the average value of the portfolio's return determines the portfolio risk. Numerically, portfolio and asset risks are formalized as the standard deviations of their returns around their means [2]. Extensions of risk formalizations are made with Value at Risk (VaR) forms that estimate parameters from probability distributions of portfolio returns [13] and/or the application of forecasts based on experts' assessments [14]. Thus, portfolio theory deals with stochastic changes in asset returns and considers two main optimization criteria: maximization of portfolio returns and minimization of risk. Portfolio theory estimates the return and risk of the portfolio over a predetermined time, for the end of the investment policy. At this point, all assets, their returns, and risks must be quantified [4]. This case imposes a strict requirement that the end of the maturity of the assets coincides with the end of the investment horizon [5]. This research differs from the above portfolio applications by adding a new requirement to the investment process by explicitly introducing a reinvestment policy to optimize the portfolio. Investment management explicitly assumes that asset maturities have different time horizons. This requirement is very strong and reinvestment policies make the portfolio problem very difficult to solve. The portfolio problem becomes combinatorial and its solution is computationally difficult. If the maturities of the assets are different, the investment policy is significantly complicated. Reinvestment requires that the proceeds arising from a final maturity period be reinvested in another asset because the investment horizon has not yet ended. A reinvestment portfolio policy is difficult to formalize in practice and applies to risk-free assets. The effectiveness of such an investment policy is assessed by the total interest from the risk-free assets and the yield at the end of the investment period. Thus, the objective of the portfolio should formalize the maximization of the sum of the asset returns [15]. Investment assets can take various forms, for example as leases in ship financing [16]. Investment strategies for the sale and purchase of ships are presented in [17].

Reinvestment decisions must take a significant amount of additional factors from business practice such as the ability to take the entrepreneurial risk, maintain liquidity conditions during the investment, and face and fulfill the growing challenges of normal business management [18]. Factoring these factors into the portfolio problem makes it very complex.

In [19], an approach to solving the reinvestment problem with assets with different maturities by applying a flexible time horizon is presented. Such a portfolio policy has different durations and this leads to different levels of return. Therefore, the investor needs to optimize the mix of portfolios, schedules, and time horizons. The portfolio problem for this case has integer variables, making optimization computationally difficult.

In [20], reinvestment portfolio considerations relate to access and use of bank loans and overdrafts. It is also concluded that the implementation of reinvestment strategies should be supported by improving the quality of the judicial system by reducing potential corruption.

The estimation of the real levels of reinvestment is discussed in [21]. It is concluded that the application of the Net Present Value as profit for the optimization does not always give real information about the efficiency of the reinvestment portfolio policy. It is recommended to apply a real economic basis such as portfolio returns to evaluate the reinvestment strategy. A similar criterion is recommended in [22] for the use of the Return On Investment (ROI) parameter. This criterion should be used to evaluate the investment policy for its profitability.

A reinvestment strategy with a flexible time horizon is studied in [19].

The survey by [23], concluded among 7000 business entities in 34 countries, allows concluding that reinvestment policies give useful results and are mainly applied by small firms compared to relatively large ones. In general, investment strategies should be derived and implemented in accordance with quantitative analysis and decision-making [24, 25].

The formalization of a portfolio problem with the implementation of a reinvestment strategy is presented in [26]. The investment period is chosen as long-term. The formalization is performed as a sequence of definitions and solutions for optimization problems that implement a short-term investment horizon. Maximization of the sum of short-term returns is the goal function of optimization.

The reinvestment policy of a business entity or enterprise must satisfy the current dynamics and expectations of cash flows. The optimization of the reinvestment must be combined with the overall activity of the company [27, 28].

This analysis of reinvestment portfolios shows that such investment strategies are quite difficult to formalize and implement. An investor should consider various assets with non-consecutive maturities; satisfaction of current business cash flows for the normal operation of the business entity; to consider flexible investment periods and last but not least to have the ability to properly manage your business without a lack of financial resources. In general, the assessment of such an investment policy is done by the portfolio return as a measure of the return on the investment. The formalization of the reinvestment leads to a complex optimization problem, the solution of which is a prerequisite for the success of the investment.

This study derives a portfolio reinvestment model. The model is used to define an optimization problem, for assets with different maturities. The assets used are risk-free and the investment is made with free cash. The solution to the optimization problem is illustrated in an EXCEL environment, which is widely applicable and beneficial for the use of the defined problem by potential users who are engaged in managing the investment of financial resources.

3. Defining the reinvestment portfolio optimization problem

The analysis made of the features of reinvestment policies insists on taking into account a set of requirements that are related to the current business management of the economic entity. In this study, assets with different maturity durations are selected as risk-free assets with three types of maturity horizons: one month, 3 months, and 6 months. Such assets are bank deposits that are offered by banking institutions.

The predefined parameters of the portfolio problem are established as follows:

- A long-term horizon of up to one year (12 months) has been established. At the end of this horizon, all assets must reach their maturity, and the income from their successive and/or simultaneous use will determine the integral of the investment return.
- The definition of the investment horizon is chosen to be longer according to the maturity periods of the assets in the portfolio.
- The company starts the investment policy with cash that is free from current business tasks.

- Regular business operations lead to limits that at the end of each month the business entity must cover with cash flows that cannot participate in the reinvestment policy.
- Business management retains its own secure resources at the end of this month with funds to meet repayment needs. These resources also cannot participate in the reinvestment, but they should be taken into account when the portfolio problem recommends the realization of new short-term investments.

The optimization problem must determine how much, what kind, and when to invest in assets with appropriate maturities. The number of investments in the relevant assets must correspond to the free available resources that satisfy the constraints on current cash flows from business operations. Therefore, available resources should be evaluated at the end of each month. These consist of current free and income coming from potential assets that have expired this month.

The portfolio problem is to determine the appropriate use of assets over the investment horizon, and its profit is to maximize the sum of all returns given by the various assets that have matured.

4. Formalization of the reinvestment problem

The portfolio optimization problem is defined as a discrete dynamic one. The discrete is chosen as one month and the time horizon is 12 months, $k = 1, \ldots, 12$. The assets have maturities of 1, 3, and 6 months. Accordingly, the solutions to the portfolio problem are denoted as:

- $u_1(k)$, for a one-month deposit;
- $u_2(k)$, for a quarterly deposit;
- $u_3(k)$, for a six-month deposit.
- The value of k determines the starting month for the duration of the maturity period of the corresponding asset.
- The interest rate for each risk-free asset is denoted by r_1, r_2, r_3.
- The minimal amount of funds to invest in deposits for categories u_1, u_2, u_3 is different and these values are denoted as m_i, i=1, 2, 3.
- For the invested resources in several assets $u_1(k)$ the income obtained at the end of the maturity period will be the multiplication $r_1\ m_1 u_1$, respectively $r_2 m_2 u_2$, and $r_3 m_3\ u_3$ – for the next types of assets.
- Available resources for reinvestment are notated as $X(k)$, $k = 0, \ldots, 12$. The value of $X(0)$ is the initial resource with which the reinvestment policy begins. The value of $X(k)$ gives the value of the resources for the beginning of each month when a new investment decision must be made.
- At the end of each month, the business entity will have resources $Y(k)$ which are the result of the maturing assets plus their income. In addition, the business entity may receive or must meet a predetermined number of payments that are scheduled to proceed from business operations or to meet loan obligations or scheduled payments. These values are known in advance and their notations are given as parameters $S(k)$ for the portfolio problem. The value $S(k^*)$ is positive when the business receives resources and negative for payments due for the specified month k^*. Thus, at the end

of each month, the received resources change depending on the due payments or receipts and the final maturities of the deposits.

- The portfolio problem includes a parameter for backup safe money E_s to be kept for the end of each month. To simplify the portfolio problem, this value is assumed to be constant, R_s = const.
- This research applies values that are currently applied in business practice in Bulgaria. Assessment of funds and income are assessed in the National currency “lev” (BGN).
- The formal description of the objective function of the portfolio problem is given as the maximization of all received revenues for the given optimization horizon or

(1) $$\max_{u_1(k),u_2(k),u_3(k)} \sum_{k=1}^{12}[r_1 m_1 u_1(k) + r_2\, m_2 u_2(k) + r_3 m_3 u_3(k)].$$

The maximization is performed against the arguments $u_i(k)$, i = 1, 2, 3; k = 1, …, 12, which gives the number of deposits, opened during the investment period. The values of $u_i(k)$ are integers and non-negative values $u_i(k) \geq 0$.

The analytical definition of the portfolio problem must meet the requirements described in Section 3 of the paper. Accordingly, the free amount of resources for each month $X(k)$ will depend on the available resources at the end of the previous month $Y(k-1)$. These relations are formally described by a set of relationship types

(2) $$X(k) = Y(k-1) + m_1 u_1(k-1) + m_2\, u_2(k-3) +$$
$$+ m_3 u_3(k-6) + r_1 u_1(k) + r_2\, u_2(k-3) + r_3 u_3(k-6) + S(k),\ k = 1, \ldots, 12.$$

This relationship states that the available resource $X(k)$ for investment at the beginning of month k is the sum of the available free resources $Y(k-1)$ of the previous month plus the sum of the one-month maturity deposits $m_i u_i(k-1)$, respectively for three months $m_2\, u_2(k-3)$, and six months $m_3 u_3(k-6)$. These relations are seen to contain time lags of 1, 3, or 6 months because they have been opened before the current time k. In addition, for deposits with a final maturity, their income is also added to the available resources $X(k)$ with the components $r_1 u_1(k)$, respectively $r_2\, u_2(k-3)$ and $r_3 u_3(k-6)$. Finally, the valuation of $X(k)$ must take into account additional resource income $S(k)$ from current business operations for a positive $S(k)$ or be reduced to cover liabilities when $S(k)$ is negative.

Thus, the estimate of free available resources for each month is formalized by a discrete-type relation with time delays and integer arguments $u_i(k)$, i = 1, 2, 3. Such a constraint in the portfolio problem demands computing power to solve such an optimization problem.

The component $Y(k)$ should estimate the resulting free resources for the end of month k. Its value is obtained by reducing the available resources at the beginning of the month $X(k)$ with the invested amount of resources for various assets $m_1 u_1(k) + m_2\, u_2(k) + m_3 u_3(k)$ and algebraically adding the predefined incoming inflows or outgoing obligations $S(k)$. The analytical descriptions of these sets of relations are

(3) $$Y(k) = M(k) - m_1 u_1(k) + m_2\, u_2(k) + m_3 u_3(k) + S(k),\ k = 1, \ldots, 12.$$

Relations (1), (2) and (3) are involved in the definition of the portfolio problem. The possibility of reinvestment is formally included in relations (2) and (3). The possibility of reinvestment policy makes the inclusion of a time delay for the available optimization resources $X(k)$. Accordingly, the free resources at the end of the month $Y(k)$ are also related to $X(k)$, making them dependent on time delays as well.

The portfolio problem should take into account that deposits lasting six months can only be opened for the first half of the investment period of one year. Accordingly, quarterly deposits should be opened by the month of October, $k = 10$, since for $k > 10$ the maturity of these assets will be above the investment horizon. These relations should be considered for (2) and (3) for different values of $k \geq 6$ and/or $k \geq 10$. To clarify these formal relations, Fig. 1 presents a graphical interpretation of the dynamic reinvestment policy.

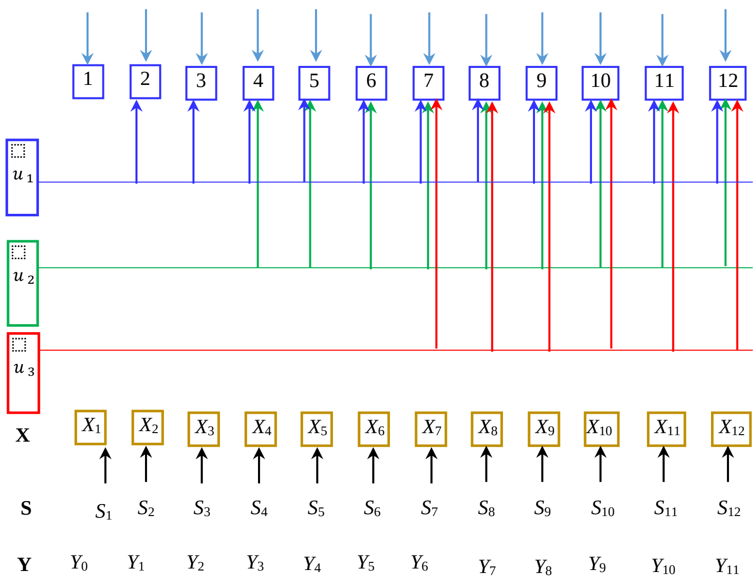


Fig. 1. Dynamical receipt of incomes from the assets

Assets u_1 with maturity 1 month can be included in the portfolio at the beginning of each month $k = 1, \ldots, 12$. Their interest is received at the beginning of months $k = 2, \ldots, 12$. The arrows in Fig. 1 illustrate the month in which the interest is available for reinvestment.

Assets u_2 with a maturity of 3 months can be included in the portfolio for $k = 1, \ldots, 10$. After month $k = 10$, this type of asset cannot be included in the portfolio because the maturity period exceeds the investment horizon of 12 months. However, they could be available for reinvestment for $k = 4, \ldots, 12$, represented as arrows in Fig. 1.

The appropriate relations are valid for asset u_3 with a maturity of 6 months. These assets can be included in the portfolio for months $k = 1, \ldots, 6$ since after this month the maturity will be outside of the investment horizon. Their interests must be available for reinvestment in months $k = 7, \ldots, 12$. In Fig. 1, month $k = 12$ is the last and the reinvestment for this month does not apply.

The analytical formulation of the portfolio problem will take the following form:

(4) $$\max_{u_1(k),u_2(k),u_3(k)} \sum_{k=1}^{12}[r_1u_1(k) + r_2\, u_2(k) + r_3u_3(k)],$$

subject to

$$X(k) = Y(k-1) + m_1u_1(k-1) + r_1m_1u_1(k),\ k = 1, \dots, 3,$$
$$X(k) = Y(k-1) + m_1u_1(k-1) + m_2\, u_2(k-3) + r_1m_1u_1(k) +$$
$$+r_2\, m_2u_2(k-3),\ k = 4, \dots, 6,$$
$$X(k) = Y(k-1) + m_1u_1(k-1) + m_2\, u_2(k-3) + m_3u_3(k-6) +$$
$$+r_1m_1u_1(k) + r_2\, m_2u_2(k-3) + r_3m_3u_3(k-6),\ k = 6, \dots, 12,$$
$$Y(k) = X(k) - (\, m_1u_1(k) + m_2\, u_2(k)\ +\ m_3u_3(k)) + S(k),\ k = 2, \dots, 12,$$
$$Y(0)=Y_0,$$
$$[r_1u_1(k) + r_2\, u_2(k) + r_3u_3(k)] \le X(k),$$
$$Y(k) \ge R,\ u_i(k),\ \ i = 1, 3;\ k = 1, \dots, 12, \text{ integer},$$
$$u_1(k) = 0\ \text{ for } 12 <\ k < 1\ ,$$
$$u_2(k) = 0 \text{ for } k \ge 10\ ,$$
$$u_3(k) = 0 \text{ for } k \ge 7\ .$$

The reinvestment policy starts with initial available free resources $Y(0)=Y_0$. The portfolio problem (4) has the form of a dynamic discrete type with integer solutions and time delays in the constraints. Its solution needs computing power and an optimization software environment. The current study aims to illustrate the solution to this problem for relatively small asset types. Here is an illustration of the solution to this problem in an Excel environment. Optimization calculations are performed with the Solver function, which is included in the Excel package.

5. Defining the parameters of the portfolio problem

The parameters that are used for the numerical definition of the portfolio problem (4) are taken from the current levels of banking policy for the levels of risk-free assets and the specific business parameters of animal husbandry for dairy and meat products from the Southern Bulgaria region.

For asset u_1 with one-month maturity, the interest rate is r_1 = 0.1% per 1 month. The minimum amount to invest in u_1 is m_1=1000 BGN.

For the asset u_2 with a three-month maturity, the interest rate is r_2 = 0.4% per maturity period. The minimum amount to invest in u_2 is m_2=2000 BGN.

For the asset u_3 with a six-month maturity, the interest rate is r_3 = 1% per maturity period. The minimum amount to invest in u_3 is m_3=3000 BGN.

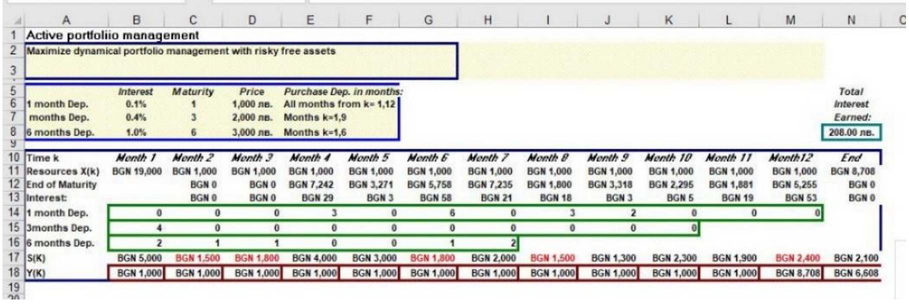
The initial investment resource is $Y(0)=Y_0$=19000 BGN.

Mandatory safe value of resources for the end of the month R_s = 1000 BGN.

The enterprise's business results for the end of each month are determined from the data set given in $S(k)$. For each month, if $S(k)>0$, these resources are added to $X(k)$ for reinvestment. For the case when $S(k)<0$, these resources reduce the amount of reinvestment $X(k)$. For this problem, the set of data is evaluated to the vector **S**=[5000; –1500; –1800; 4000; 3000; –1800; 2000; –1500; 1300; 2300; 1900; –2400; 2100].

6. Programming the Excel sheet

The solution to the defined portfolio problem (4) is obtained by appropriate programming in an Excel environment as a widely available software product. The working interface of the Excel sheet is given in Fig. 2.



	A	B	C	D	E	F	G	H	I	J	K	L	M	N
1	Active portfoliio management													
2	Maximize dynamical portfolio management with risky free assets													
3														
5		Interest	Maturity	Price	Purchase Dep. in months:									Total
6	1 month Dep.	0.1%	1	1,000 лв.	All months from k= 1,12									Interest
7	months Dep.	0.4%	3	2,000 лв.	Months k=1,9									Earned:
8	6 months Dep.	1.0%	6	3,000 лв.	Months k=1,6									208.00 лв.
10	Time k	Month 1	Month 2	Month 3	Month 4	Month 5	Month 6	Month 7	Month 8	Month 9	Month 10	Month 11	Month12	End
11	Resources X(k)	BGN 19,000	BGN 1,000	BGN 1,000	BGN 1,000	BGN 1,000	BGN 1,000	BGN 1,000	BGN 1,000	BGN 1,000	BGN 1,000	BGN 1,000	BGN 1,000	BGN 8,708
12	End of Maturity		BGN 0	BGN 0	BGN 7,242	BGN 3,271	BGN 5,758	BGN 7,235	BGN 1,800	BGN 3,318	BGN 2,295	BGN 1,881	BGN 5,255	BGN 0
13	Interest:		BGN 0	BGN 0	BGN 29	BGN 3	BGN 58	BGN 21	BGN 18	BGN 3	BGN 5	BGN 19	BGN 53	BGN 0
14	1 month Dep.	0	0	0	3	0	6	0	3	2	0	0	0	
15	3months Dep.	4	0	0	0	0	0	0	0	0	0			
16	6 months Dep.	2	1	1	0	0	1	2						
17	S(K)	BGN 5,000	BGN 1,500	BGN 1,800	BGN 4,000	BGN 3,000	BGN 1,800	BGN 2,000	BGN 1,500	BGN 1,300	BGN 2,300	BGN 1,900	BGN 2,400	BGN 2,100
18	Y(K)	BGN 1,000	BGN 1,000	BGN 1,000	BGN 1,000	BGN 1,000	BGN 1,000	BGN 1,000	BGN 1,000	BGN 1,000	BGN 1,000	BGN 1,000	BGN 8,708	BGN 6,608

Fig. 2. Excel sheet working interface

The notations and components of the portfolio problem (4) are interpreted graphically as follows.

The number of assets u_1 is given in cells B14:M14 for $k = 1, \ldots, 12$.

The number of assets u_2 is given in cells B15:K15 for $k = 1, \ldots, 10$.

The number of assets u_3 is given in cells B16:H16 for $k = 1, \ldots, 6$.

The cells of this set contain the number of relevant assets and when they were have been included in the portfolio as reinvestments.

The interest rate of assets is given in cells B6:B8 containing the values **r** = [0.1; 0.4; 1]. The minimum amount of resources for each asset is given in cells D6:D8, containing **m = [**1000; 2000; 3000]. The data set $S(k)$ is given in cells B17:M17. The value of $S(k)$ in red is negative and it reduces the current reinvestment resource $X(k)$.

The cells of row 11 with cells B11:M11 contain the free resources for reinvestment of the previous month $Y(k-1)$. The value for the first month is the initial free resource $Y(0) = Y_0 = 19000$ BGN is given in cell B11 and this value is the initial free resource $X(1)$ for investment.

The cells in row 12 estimate the resources received from the assets with a final maturity. Cells B14:D14 correspond to months $k = 1, 2, 3$ and for this period only one-month assets can add recovery interest. The valuation of these resources for reinvestment is $m_1 u_1(k-1)$. For illustration, the code for cell D14 is “\$D\$6×C14”.

For months $k = 4, 5$ end of maturity there may be three-month asset. Their resource should be added to the potential of the one-month assets in the ratio $m_1 u_1(k-1) + m_2\, u_2(k-3)$. For illustration, the relation in cell F12 is

\$D\$6× E14+ \$D\$7×C15.

For months $k = 6, \ldots, 12$ all three types of assets can have maturities, and the corresponding resources, which can be used for reinvestment are the sum $m_1 u_1(k-1) + m_2\, u_2(k-3) + m_3 u_3(k-6)$. For illustration, this relation is coded in cell M12 like

\$D\$6×L14+\$D\$7×J15+\$D\$8×H16.

The cells in row 13 estimate the interest rates that are earned on assets with a final maturity. For months $k = 2, 3$ only one-month assets can give interest and their value is given by the relation $r_1 m_1 u_1(k)$. For illustration, cell D13 contains the relation

$B$6×$D$6×C14.

For $k = 4, 5$ the quarterly assets can generate an additional interest rate according to the sum $r_1 m_1 u_1(k) + r_2\, m_2 u_2(k-3)$. For illustration, cell F13 contains the relation

$B$6×$D$6×E14+$B$7×$D$7×C15.

For $k = 6, \ldots, 12$ the interest rates can be generated from all three types of assets. The resulting value is

$$r_1 m_1 u_1(k) + r_2\, m_2 u_2(k-3) + r_3 m_3 u_3(k-6).$$

For illustration, cell M13 contains the relation

$B$6×$D$6×L14+$B$7×$D$7×J15+$B$8×$D$8×H16.

Available free resources for reinvestment are generally estimated as

$$Y(k-1) + m_1 u_1(k-1) + m_2\, u_2(k-3) + m_3 u_3(k-6) + r_1 m_1 u_1(k) + \\ + r_2\, m_2 u_2(k-3) + r_3 m_3 u_3(k-6),$$

where $Y(k-1)$ is the free returns after reinvestment of the previous month $k - 1$. The new reinvestment is performed with the value

$$m_1 u_1(k) + m_2\, u_2(k) \; + \; m_3 u_3(k).$$

Finally, the business liabilities $S(k)$ are added algebraically. Thus, the free resources remaining after the reinvestment are

$$Y(k) = X(k) - \left(m_1 u_1(k) + m_2\, u_2(k) \; + \; m_3 u_3(k) \right) + S(k).$$

These estimates are given on row 18 and for illustration cell M18 contains

SUM(M11:M13)-SUMPRODUCT(M14:M16, $D$6:$D$8)-M17.

The calculation

SUMPRODUCT(B14:B16;$D$6:$D$8)

gives the value of the reinvestment resource allocated to a new asset.

The SUMPRODUCT(.) function does a vector multiplication $m_1 u_1(k) + m_2\, u_2(k) \; + \; m_3 u_3(k)$ between the number of assets $u_1(k)$, $u_2(k)$, $u_3(k)$ and their required quantities $m_1(k)$, $m_2(k)$, $m_3(k)$.

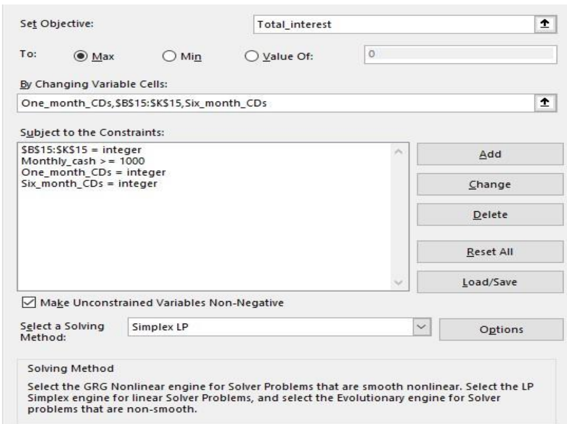


Fig. 3. Coding illustration of the Solver function

The objective function is the sum of the interest rates that are calculated in row 13. The objective cell is placed in cell N8 and contains the relation "SUM(B13:M13)".

The solution to the problem (4) is performed by the software application Solver. In its window, the objective function is placed in the "Set Objective" field. Variables are placed in the "By changing variable sets" box. Constraints are entered in the "Subject to the constraint" field. In the last field, constraints for non-negative and integer values of the solutions $u_1(k),\ u_2(k),\ u_3(k)$ are added. The minimum value of available resources after reinvestment is determined at $X(k) \geq R_s$=1000 BGN. An illustration of the command window of Solver is given in Fig. 3.

7. Numerical simulations and evaluations

The program codes for solving the portfolio reinvestment problem (4) allow its solution to be performed with several changes in the problem parameters. This study analyzes the impact of asset investment levels on the number of reinvestments over the investment horizon of one year. Reinvestment may create difficulties from an administrative, organizational, and/or financial point of view if each reinvestment is associated with additional tax payments. Analysis of the relationship between interest rates and the number of reinvestments can provide suggestions to the decision maker for choosing a preferred type of asset to be recommended for inclusion in an active reinvestment policy portfolio.

The reinvestment problem (4) here is solved by several changes in the interest rate on the three-month risk-free assets. The solutions to the problems are presented in Table 1.

Table 1. Analysis of the influence of the interest rate on r_2

r_1, %	r_2, %	r_3, %	u_1, number	u_2, number	u_3, number	Reinvestments $u_1+u_2+u_3$	Total interest, BGN
0.1	0.3	1	36	0	6	42	200.65
0.1	0.4	1	14	4	7	25	208
0.1	0.5	1	14	4	7	25	215.36
0.1	0.6	1	1	18	1	20	235.89
0.1	0.7	1	1	18	1	20	269.90
0.1	0.8	1	7	19	0	26	304.38
0.1	0.9	1	7	19	0	26	341.88
0.1	1	1	8	19	0	27	379.48

The graphical interpretation of these results is given in Fig. 4.

These results show that the spreads between asset interest rate levels play an important role in the number of assets that are included in the reinvestment portfolio. It can be seen that an increase in the rate r_2 from 0.3% to 1% as r_3 leads to a significant increase in the number of quarterly assets u_2. Accordingly, the six-month u_3 assets decline, as their rate is very close to the three-month u_2. The portfolio problem (4) for reinvestment prioritizes short reinvestment durations when the corresponding interest rates are close in value.

The portfolio problem (4) can be used not only to define the current reinvestment decision, but also to evaluate the influence of rates, the amount of monthly cash required for safe maintenance, the minimum value of investment resources *m*, and other parameters of the reinvestment.

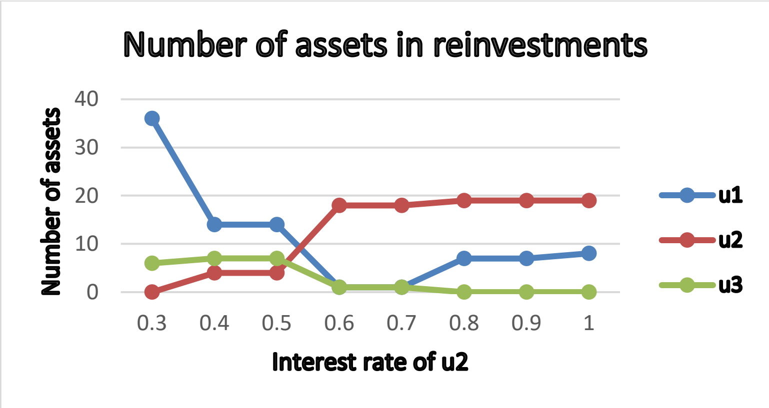


Fig. 4. Relationship between the interest rate r_2 and the number of assets in reinvestments

This research derives a risk-free asset reinvestment model. The model is applied to define an optimization problem that plans investment and reinvestment actions for portfolio management. The purpose of such management is to reinvest the income from the short-term maturity of the assets. Difficulties arising from reinvestment come from the different maturities of the assets. Reinvestment aims at optimal use of the dynamics of cash flows of the business entity. This supports the increase in wealth of the investor. Accordingly, this positively affects the normal operating activities of the business.

This research develops a quantitative approach to assess the benefits of reinvestment. The defined optimization problem is in a discrete type dynamic optimization problem. It takes into account many constraints and requirements that are practically applicable in running a business. An advantage of this defined optimization problem is that it can be formalized for solution in a well-known software environment such as Excel. However, if the dimensionality of the task increases with the number of assets and/or the length of the investment horizon is longer, it is natural to use other software and computing environments, with higher computational performance.

The problem has the potential to involve additional considerations that deal with the risky nature of the investment process. In that case, additional relationships targeting risk and covariance between asset returns must be included in the reinvestment optimization problem. This is a potential direction for follow-up and future research on the topic of asset reinvestment with different maturities.

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