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Optimization of Characteristics for a Stochastic Agent-Based Model of Goods Exchange with the Use of Parallel Hybrid Genetic Algorithm

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Abstract**:** *A novel approach to modeling stochastic processes of goods exchange between multiple agents is presented, considering the possibility of optimizing the environment's characteristics and individual decision-making strategies. The proposed model makes it possible to form optimal states when choosing the moments of concluding barter and monetary transactions at the individual level of each agent maximizing the utility function. A new parallel hybrid Real-Coded Genetic Algorithm and Particle Swarm Optimization (RCGA-PSO) has been developed, combining methods of evolutionary selection based on well-known heuristic operators with methods of swarm optimization and machine learning. The algorithm is characterized by the best time efficiency and accuracy in comparison with other methods. The software implementation of the developed algorithm and model has been performed using the FLAME GPU framework. The possibility of using the RCGA-PSO Algorithm to optimize the characteristics of the environment and strategies for making individual decisions by agents involved in barter and monetary interactions is demonstrated.*

Keywords**:** *Stochastic simulation model; agent-based modeling of barter and monetary interactions; optimal control in agent models; real-coded genetic algorithms; machine learning methods; particle swarm optimization; FLAME GPU.*

1. Introduction

In modern times, there is a significant complication of the requirements for economic planning Decision Support Systems (DSS), which is due to the need to find the best scenarios for control of complex multiagent socio-economic systems. Among such systems, more attention should be paid to models of economic dynamics based on the equilibrium of forecasts of economic agents [1, 2], models of random sales [3], and models of the behavior of producers and consumers interacting at random moments [4, 5].

As a rule, in the above multi-agent systems, the description of the interaction between sellers and buyers is built as a controlled Markov process [6, 7]. One of the issues explored with this approach is the role of money in the economy and the existence of barter trade. The same construction can be applied to situations where there are several variants of money. The model is based on the interaction between random pairs of agents at random times with their mutual consent. At the same time, agents can be in a state of readiness or refusal to participate in transactions, thereby implementing their trading strategy [2]. The existence of optimal strategies in Markov processes with a finite number of states has been theoretically proven [7] However, finding the best individual strategies is a difficult problem of optimal control. At the same time, it is impossible to use traditional dynamic programming methods based on the construction and solution of the Bellman equation to determine the optimal strategies of multiple interacting economic agents [2]. Therefore, this paper proposes a novel approach using a hybrid genetic algorithm.

In the past, Real-Coded Genetic Algorithms (RCGAs) have been developed and applied to optimize the characteristics of the environment in a multisector bounded-neighborhood model [8]; to optimize the characteristics of the ecological and economic system [9]; to improve the maneuverability of unmanned vehicles on the scale of the 'smart city' [10, 11], to minimize a simplified model of the energy function of the molecule [12]; for grasping an object of a priori unknown shape [13] and others. For the first time, the use of heuristic real-coded operators was proposed in [14], which made it possible to apply genetic algorithms (of the RCGA class) to solve high-dimensional optimization problems. On the other hand, there is a well-known problem of high computational complexity of RCGA, which is observed when solving optimization problems, the objective functionals of which are calculated because of simulation modelling [9]. Therefore, in this paper, an attempt is made to improve the time efficiency of the genetic algorithm, mainly due to integration with the Particle Swarm Optimization Algorithm (PSO Algorithm) [15, 16] and the use of an artificial neural network designed and trained to quickly approximate the objective functional in the optimization problem under consideration. The advantage of PSO is a significantly faster ("swarm") mechanism for searching for promising solutions, which does not require the performance of resource-intensive evolutionary search operations used in RCGA. However, swarm algorithms [16] often converge at local extrema and give fewer stable solutions. In addition, it is known [17], that the use of ANN and some other machine learning methods (such as the support vector machine, regression based on Gaussian processes, etc.) allows the fitness-function approximation to reduce the total number of recalculations with the original simulation model.

The purpose of this article is to develop a new approach to modeling stochastic processes of goods exchange between multiple agents, considering the possibility of optimizing the environment's characteristics and the decision-making of agents using the proposed heuristic technique that combines the Real-Coded Genetic Algorithm and Particle Swarm Optimization (RCGA-PSO). Within the approach, a software implementation of the proposed heuristic algorithm, aggregated by objective functions with the developed stochastic model of the exchange of goods between multiple agents and its ANN-based surrogate model, has been completed using the FLAME GPU supercomputer simulation framework [18-20].

2. Stochastic model of goods exchange between multiple agents

2.1. Model concept

The initial statement of the problem is presented in [2]. A system is considered in which paired interactions are realized between multiple agents (sellers and buyers) at random moments of time, provided that their mutual interests coincide (i.e., there is a supply and demand for a particular product). Each agent consistently produces, exchanges, and consumes a unit volume of one of the fixed sets of products. After the consumption of the obtained product, the production of another begins. At random moments, a unit volume of a new product is generated at an agent in the state of production, which is then transferred or sold to a buyer interested in this particular product. Transactions between agents are carried out, provided that they are in a certain visibility zone, determined by the radius of trade interaction 100×100. In this case, the radius of the trade interaction is determined by the range of cells considered neighboring [8].

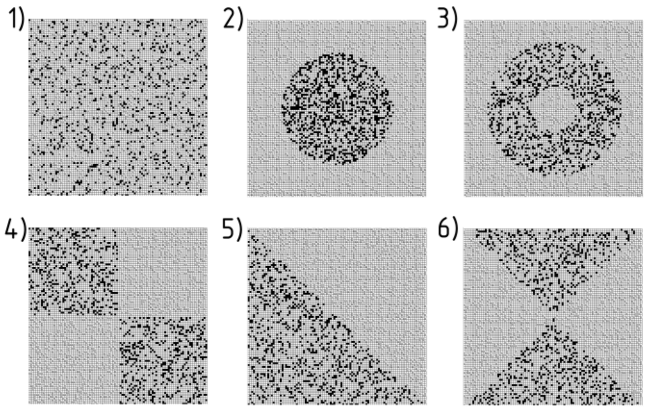

Fig. 1. Different scenarios for the initial spatial distribution of agents

There are not any deals if the pair of agents with money are met. If the agent-buyer has money, and the agent-seller does not have money, but there is a product the buyer needs, and both agents are in a state of readiness for monetary transactions, then the corresponding product is transferred from the seller to the buyer for a monetary reward. If the agent-buyer and the agent-seller do not have money, but they have products that each other needs and both agents are in a state of readiness for barter deals, then the corresponding products are exchanged. With the successful completion of monetary and barter deals, the individual utility function of each interacting agent (both the seller and the buyer) is incremented by an amount depending on the distance between the target and the purchased product. At the same time, the seller transitions to the state of waiting for the production of a new product, within which the sets of available and target products are updated (that is, the characteristics of the supply and demand of this agent change). For simplicity, in the considered multi-agent system, the time costs for the production of new products are not considered, i.e., agent-sellers produce their new goods immediately after the conclusion of deals.

An important difference between the proposed model and the previously known ones [2, 6, 7] is the consideration of the influence of the spatial distribution of agents on the dynamics of trade interactions. In particular, various options for the initial placement of agents in a discrete space with a limited dimensionality are considered (Fig. 1).

In Fig. 1, the following denotations are used:

- the "uniform distribution" of sellers (seller agents are marked in black, and buyer agents – in grey);
- the "ring distribution" with a core consisting of seller agents, surrounded by agents-buyers;
- "torus-shaped distribution", providing alternation of buyers and sellers from the center to the periphery;
- "chess distribution", providing placement of buyers and sellers in their sectors of an equal area;
- "diagonal distribution";
- "cone-shaped distribution".

Initially, the population is divided into two conditional groups: sellers with their own sets of products, but who do not have money, and buyers, who have money, but do not own products. During the simulation period, agents can change their roles many times (i.e., sellers become buyers and vice versa depending on whether they have money, products, etc.). They can also move in a discrete space with a given probability and in a given (fixed) range, which affects the dynamics of interactions and transactions between agents.

2.2. Model description

A system is considered in which, at each moment of model time, a barter or monetary transaction can be carried out between each pair of agents that are in the visibility range (i.e., one transaction between a certain pair of interacting agents at each moment of time).

The strategy for making individual decisions by agents consists of the formation of optimal states (readiness) for concluding trade transactions for each moment of time. Such states are set outside the model using the developed hybrid genetic algorithm RCGA-PSO. The remaining states related to the choice of types of transactions, etc., are endogenous.

Here:

- $T = \{t_0, t_1\ ...,\ |T|\}$ is the set of time moments (by days), $|T|$ is the total number of time moments; $t_0 \in T$ and $t_{|T|} \in T$ are the initial and final moments of the model;

- $I = \{i_1, i_2, ..., i_{|I|}\}$ is the set of indices of agents, where $|I|$ is the total number of agents, $\tilde{i} \in I$ is the sellers' indices, $\hat{i} \in I$ is the buyers' indices;
- $P = \{p_1, p_i, ..., p_{|P|}\}$ is the set of indices of products; $|P|$ is the total number of products; $p_i(t_k) \in P,\ i \in I,\ t_k \in T,$ is the indices of products that is available to the i-th agent; $d_i(t_k) \in P,\ i \in I,\ t_k \in T,$ is the index of the product that is needed by the i-th agent;
- $\{b_i(t_k),\ m_i(t_k)\} \in \{0,\ 1\},\ i \in I,$ is the readiness state of the i-th agent to conclude barter and monetary deals, respectively, at the moment t_k $(t_k \in T)$: 0 – the transactions are prohibited, 1 – the transactions are allowed;
- $\pi_i(t_{k-1}),\ i \in I,$ is the amount of available money of the i-th agent at the moment t_{k-1} $(t_{k-1} \in T)$;
- $v_{p_i}(t_{k-1}) \in \{0,\ 1\},\ p_i \in P,\ i \in I,$ is the available quantity of the p_i -th product of the i-th agent at the moment t_{k-1} $(t_{k-1} \in T)$;
- $u_i(t_{k-1}),\ i \in I,$ is the value of the utility function of the i-th agent at the moment t_{k-1} $(t_{k-1} \in T)$;
- $\delta_{\tilde{i}\hat{i}}(t_k)$ is the distance between the $p_{\tilde{i}}(t_k)$-product $(p_{\tilde{i}}(t_k) \in P)$ belonging to the $\tilde{i}$-th agent-seller $(\tilde{i} \in I)$ and the $d_{\hat{i}}(t_k)$-product $(d_{\hat{i}}(t_k) \in P)$ belonging to the $\hat{i}$-th agent-buyer $(\hat{i} \in I)$, measured along the length of the arc of a numerical circle with evenly distributed numbers at the moment t_{k-1} $(t_{k-1} \in T)$,

(1) $$\delta_{\tilde{i}\hat{i}}(t_k) = \frac{1}{|P-1|} \min\left\{ \left|p_{\tilde{i}}(t_k) - d_{\hat{i}}(t_k)\right|,\ |P| - \left|p_{\tilde{i}}(t_k) - d_{\hat{i}}(t_k)\right| \right\};$$

- $\alpha \geq 0$ is the coefficient of contractuality (i.e., threshold compliance) of the product of the agent-seller with the interests of the agent-buyer; because of the $\alpha \in [0,\ 1]$, the coefficient can be interpreted as the probability of a trade.

Then, the assessment of the level of compliance of the product of the agent-seller with the interests of the agent-buyer can be given as

(2) $$\beta_{\tilde{i}\hat{i}}(t_k) = \begin{cases} 1 \text{ if } \delta_{\tilde{i}\hat{i}}(t_k) \leq \alpha, \\ 0 \text{ if } \delta_{\tilde{i}\hat{i}}(t_k) > \alpha. \end{cases}$$

The states of readiness of the i-th agent $(i \in I)$ to the conclusion of barter and monetary transactions are set for each moment of time t_k $(t_k \in T)$ using lognormal distributions with given characteristics:

(3) $$b_i(t_k) = \begin{cases} \dfrac{\left\lfloor \ln N(\mu_b,\ \sigma_b^2) \right\rfloor}{\left\lfloor \ln N(\mu_b,\ \sigma_b^2) \right\rfloor} \text{ if } \ln N(\mu_b,\ \sigma_b^2) > 0, \\ 0 \text{ if } \left\lfloor \ln N(\mu_b,\ \sigma_b^2) \right\rfloor = 0, \end{cases}$$

$$(4)\qquad m_i(t_k)=\begin{cases}\dfrac{\lfloor \ln N(\mu_m,\ \sigma_m^2)\rfloor}{\lfloor \ln N(\mu_m,\ \sigma_m^2\ )\rfloor} \text{ if } \ln N(\mu_m,\ \sigma_m^2)>0,\\ 0 \text{ if } \lfloor \ln N(\mu_m,\ \sigma_m^2)\rfloor=0,\end{cases}$$

where $\ln N(\mu_b,\ \sigma_b^2)$, $\ln N(\mu_m,\ \sigma_m^2)$ are random values having lognormal distributions with parameters $\mu_b,\ \sigma_b^2$ and $\mu_m,\ \sigma_m^2$, respectively.

At each moment t_k $(t_k \in T)$, between the $\tilde{i}$-th agent-seller $(\tilde{i} \in I)$ and the $\hat{i}$ -th agent-buyer $(\hat{i} \in I)$ a monetary or barter transaction can be done, the result of which is a symmetrical change in the amount of money (during monetary interaction) and the products available to agents, i.e.,

$$(5)\qquad \pi_{\tilde{i}}(t_k)=\begin{cases}\pi_{\tilde{i}}(t_{k-1})+ \text{ if I is fulfilled,}\\ \pi_{\tilde{i}}(t_{k-1}) \text{ if III is fulfilled,}\end{cases}$$

$$(6)\qquad \pi_{\hat{i}}(t_k)=\begin{cases}\pi_{\hat{i}}(t_{k-1})-1 \text{ if I is fulfilled,}\\ \pi_{\hat{i}}(t_{k-1}) \text{ if III is fulfilled,}\end{cases}$$

$$(7)\qquad v_{p_{\tilde{i}}}(t_k)=\begin{cases}v_{p_{\tilde{i}}}(t_{k-1})-1 \text{ if I or II is fulfilled,}\\ v_{p_{\tilde{i}}}(t_{k-1}) \text{ if III is fulfilled,}\end{cases}$$

$$(8)\qquad v_{p_{\hat{i}}}(t_k)=\begin{cases}v_{p_{\hat{i}}}(t_{k-1})+1 \text{ if I or II is fulfilled,}\\ v_{p_{\hat{i}}}(t_{k-1}) \text{ if III is fulfilled.}\end{cases}$$

Here I, II and III are the next three conditions.

I. $\pi_{\tilde{i}}(t_{k-1})=0$ and $\pi_{\hat{i}}(t_{k-1})>0,$ and $\beta_{\tilde{i}\hat{i}}(t_k)=1,$ and $m_{\tilde{i}}(t_k)m_{\hat{i}}(t_k)=1$, which means that the agent-seller has no money, the agent-buyer has money, the seller has a product, the threshold compliance of the product of the agent-seller with the interest of the agent-buyer, and the mutual readiness of agents for monetary deals. Consequently, monetary interaction is realized between these agents.

II. $\pi_{\tilde{i}}(t_{k-1})=0$ and $\pi_{\hat{i}}(t_{k-1})=0,$ and $\beta_{\tilde{i}\hat{i}}(t_k)=1,$ and $b_{\tilde{i}}(t_k)b_{\hat{i}}(t_k)=1$, which means that the agent-seller and the agent-buyer have no money, the threshold correspondence of the product of the agent-seller to the interest of the agent-buyer, and the mutual willingness of the agents to barter deals. Consequently, barter interaction is realized between these agents.

III. $(\pi_{\tilde{i}}(t_{k-1})>0$ and $\pi_{\hat{i}}(t_{k-1})>0)$ or

$(\pi_{\tilde{i}}(t_{k-1})=0$ and $\pi_{\hat{i}}(t_{k-1})>0,$ and $p_{\tilde{i}}(t_k)=d_{\hat{i}}(t_k),$ and $m_{\tilde{i}}(t_k)m_{\hat{i}}(t_k)=0)$ or

$(\pi_{\tilde{i}}(t_{k-1})=0$ and $\pi_{\hat{i}}(t_{k-1})=0,$ and $p_{\tilde{i}}(t_k)=d_{\hat{i}}(t_k),$ and $b_{\tilde{i}}(t_k)b_{\hat{i}}(t_k)=0)$ or $\beta_{\tilde{i}\hat{i}}(t_k)=0$, which means the meeting of two agents with money or the unreadiness of agents for a deal, or the threshold discrepancy between the product of the agent-seller and the interest of the agent-buyer. Therefore, there is no interaction between these agents.

The value of the utility function of the i-th agent $(i \in \{\tilde{i} : \hat{i} \in I,\ \beta_{\tilde{i}\hat{i}}(t_k) = 1\})$ at the moment is calculated as

(9) $$\Delta_i(t_k) = \beta_{\tilde{i}\hat{i}}(t_k)\left((\delta_{\tilde{i}\hat{i}}(t_k)+1)^{-\nu} - \lambda r\right).$$

Here:

- $r \in [1,\ \bar{r}]$ is the radius of trade interaction, i.e., the range of cells of the discrete placement space of agents considered to be neighbors (see Fig. 1), $\bar{r}$ is the maximum allowable distance between interacting agents;
- $\{v,\ \lambda\}$ are the coefficients that determine the impact of the costs of the distance between the target and the purchased product, as well as between the buyer and the seller, respectively; the values of these coefficients are chosen in such a way that the condition $\Delta_i(t_k) \geq 0\ \forall\ i \in I,\ t_k \in T$, is observed, for example, $v = 1.5,\ \lambda = 0.01$ and $\bar{r} = 20$.

To form optimal decision-making strategies for the i-th agent $(i \in I)$ it is possible (with some assumption) to replace individual utility functions with the average utility of future consumption for an ensemble of agents, which can be defined as

(10) $$U = \frac{1}{|I|}\sum_{t_k=0}^{|T|}\sum_{i=1}^{|I|}\Delta_i(t_k).$$

In this case, it is possible to determine the following control parameters for the multi-agent system being studied:

- $\mu_b, \mu_m \in [-1,\ 1]$, and $\sigma_b^2, \sigma_m^2 \in (0,\ 1]$ are the parameters of log-normal distributions that determine the readiness of agents to conclude barter and monetary transactions;
- $c \in \{1,\ 2, 3, 4, 5, 6\}$ is the configuration of the initial distribution of agents in a discrete space (see Fig. 1);
- $\alpha \in [0,\ 1]$ is the coefficient of contractuality (i.e., threshold compliance) of the product of the agent-seller with the interests of the agent-buyer, which determines the probability of a deal;
- $r \in [1,\ \bar{r}]$ is the radius of trade interaction;
- $h \in [0,\ 1]$ is the probability of moving agents in the discrete space.

Then, the problem of seeking optimal strategies for the agents' behavior and improving the environment's characteristics can be formulated in the following form.

Problem A. Maximize the average utility of future consumption for an ensemble of agents over the sets of control parameters that determine the states of agents $\{\mu_b,\ \sigma_b^2,\ \mu_m,\ \sigma_m^2\}$ and environmental characteristics $\{c,\ \alpha,\ r,\ h\}$:

(11) $$\max_{\{\mu_b,\, \sigma_b^2,\, \mu_m,\, \sigma_m^2\},\, \{c,\, \alpha,\, r,\, h\}} U,$$

s.t.

$\mu_b, \mu_m \in [-1,\ 1],\ \sigma_b^2, \sigma_m^2 \in (0,\ 1],\ \ c \in \{1,\ 2, 3, 4, 5, 6\},\ \ \alpha \in [0,\ 1],\ \ r \in [1,\ \bar{r}],\ h \in [0,\ 1].$

The developed model has been implemented in the FLAME GPU supercomputer agent-modeling environment [18]. This model is aggregated through the objective function (10) with the proposed genetic RCGA-PSO Algorithm for solving **Problem A.**

3. Hybrid genetic optimization algorithm

3.1. Description of RCGA-PSO

A novel hybrid genetic optimization RCGA-PSO Algorithm has been developed, aggregated by the target functional with the proposed stochastic model for the exchange of goods. The developed RCGA-PSO Algorithm is shown in Fig. 2.

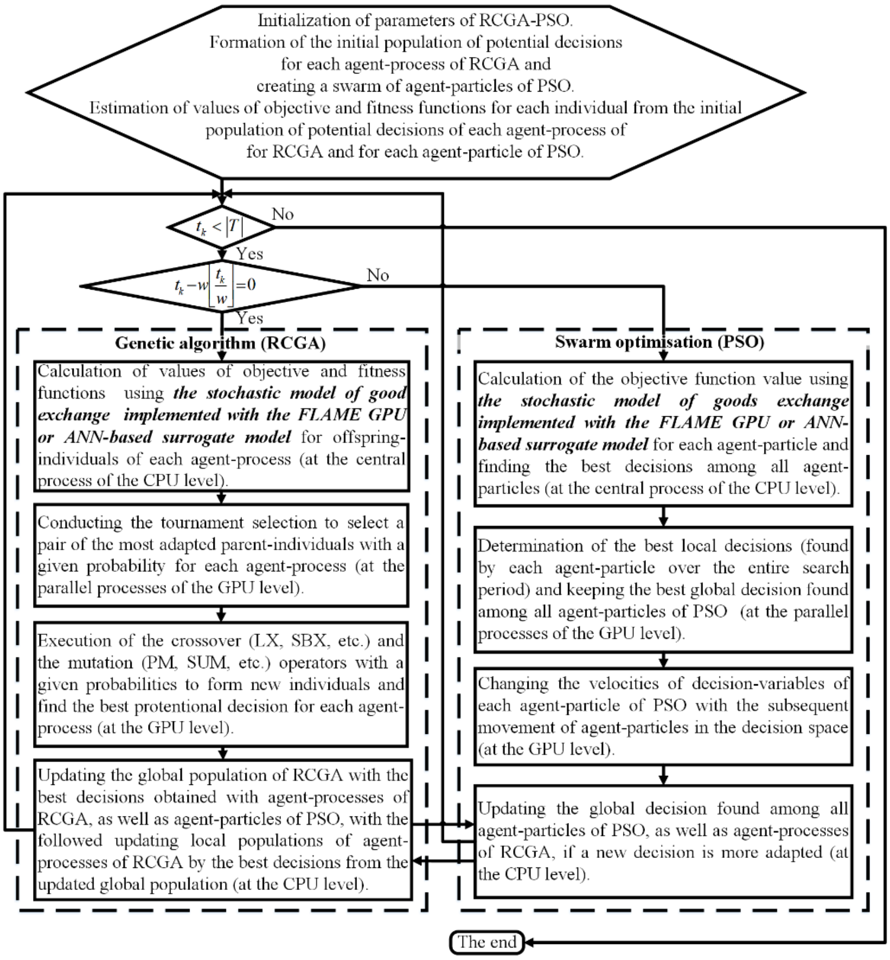

Fig. 2. Developed hybrid genetic RCGA-PSO Algorithm

As shown in Fig. 2, the functional logic of the proposed algorithm is effectively distributed between the Central Processing Unit (CPU) and parallel streams of the Graphics Processing Unit (GPU).

The advantage of using a genetic algorithm is the possibility of a relatively fast evolutionary search for suboptimal strategies for making individual decisions by agents and environmental parameters that ensure the maximization of the objective function which is the utility of future consumption for an ensemble of agents. A similar problem belongs to the class of simulation-based optimization when the value of the objective and fitness functions is calculated as a result of simulation modeling [9-11]. Another important advantage of this approach is the ability to use both the CPU and GPU system architecture.

A feature of RCGA-PSO is the combination of an evolutionary search procedure based on real-coded heuristic operators (selection, crossover, and mutation) – RCGA with a parallel swarm optimization algorithm PSO to achieve the highest performance.

In Fig. 2, the following variables and notations are used:

- $t = \{t_0, t_1, ..., |T|\}$ is the set of iterations of the Genetic Algorithm (GA), $|T|$ is the total number of iterations in GA, $t_0 \in T$, $t_{|T|} \in T$ are the initial and final moments of GA;
- w is the frequency of alternating the use of RCGA and PSO to search for the best potential solutions;
- LX (Laplace Crossover), SBX (Simulated Binary Crossover), PM (Power Mutation), SUM (Scalable Uniform Mutation) are known crossover and mutation operators [9].

The main characteristic of the RCGA-PSO (common for algorithms of the RCGA's classes) is the fitness function, which is calculated for each individual of each agent-process at iteration t_k $(t_k \in G)$:

(12)
$$\hat{f}_{j_i}(\mathbf{x}_{j_i}(t_k)) = \frac{f_{j_i}(\mathbf{x}_{j_i}(t_k))}{\left|\sum_{j=1}^{|J_i|} f_{j_i}(\mathbf{x}_{j_i}(t_k))\right|}.$$

Here:

- $I = \{i_1, i_2, ..., i_{|I|}\}$ is the set of indices of agent-processes of RCGA, where $|I|$ is the total number of agent-process;
- $J_i = \{j_{i1}, j_{i2}, ..., j_{|J_i|}\}$ is the set of indices of individuals (consisting of the vector of values of the decision-variables and the value of the objective function) belonging to the *i*-th agent-process of RCGA $(i \in I)$, where $|J_i|$ is the total number of individuals in the population of potential solutions;
- $\mathbf{x}_{j_i}(t_k) \in [\underline{\mathbf{x}}, \overline{\mathbf{x}}]$ is the vector of the decision-variables' values at the moment t_k $(t_k \in T)$, belonging to the j_i-th individual $(j_i \in J_i)$ of the *i*-th agent-

process of RCGA $(i \in I)$ at the moment t_k $(t_k \in T)$, where $\underline{\mathbf{x}}, \bar{\mathbf{x}}$ are the lower and upper boundary values of the decision-variables;

- $f_{j_i}(\mathbf{x}_{j_i}(t_k)),\ i \in I,\ j_i \in J_i$, is the value of the objective function calculated for each j_i -th individual $(j_i \in J_i)$ of the i-th agent-process of RCGA $(i \in I)$ at the moment t_k $(t_k \in T)$ using the developed stochastic model of goods exchange.

When is using the PSO Algorithm in RCGA-PSO, the velocity vector for the decision variables is calculated, which determines the position of the i-th agent-particle $(i \in I)$, in the space of potential decisions at the moment t_k, $(t_k \in T,\ k = 1,\ 2,\ ...,\ K)$:

(13) $$\mathbf{v}_i(t_k) = \theta \mathbf{v}_i(t_{k-1}) + c_1 q(0,\ 1)(\mathbf{x}_i^*(t_{k-1}) - \mathbf{x}_i(t_{k-1})) + c_2 e(0,\ 1)(\mathbf{x}^g(t_{k-1}) - \mathbf{x}_i(t_{k-1})),$$

(14) $$\mathbf{x}_i(t_k) = \begin{cases} \mathbf{x}_i(t_{k-1}) + \mathbf{v}_i(t_{k-1}) \text{ if } \mathbf{x}_i(t_{k-1}) + \mathbf{v}_i(t_{k-1}) \in [\underline{\mathbf{x}},\ \bar{\mathbf{x}}], \\ \mathbf{x}_i(t_{k-1}) \text{ if } \mathbf{x}_i(t_{k-1}) + \mathbf{v}_i(t_{k-1}) \notin [\underline{\mathbf{x}},\ \bar{\mathbf{x}}]. \end{cases}$$

Here:

- $I = \{i_1,\ i_2,\ ...,\ i_{|I|}\}$ is the set indices of agent-particles of PSO, where $|I|$ is the total number of agent-particles;
- $\mathbf{x}_i^*(t_{k-1})$, $\mathbf{x}^g(t_{k-1})$ are the best potential decisions obtained by the agent-particles of PSO during the search period and all agent-particles at the moment t_k $(t_{k-1} \in T)$;
- $q(0,\ 1),\ e(0,\ 1)$ are the random values uniformly distributed on the interval $[0,\ 1]$;
- $\theta,\ c_1,\ c_2$ are constants, the values of which, as a rule, are set in the following ranges: $\theta \in [0.4,\ 1.4],\ c_1 \in [1.5,\ 2],\ c_2 \in [2,\ 2.5]$.

The interaction between RCGA and PSO is provided by the periodic exchange of the best potential decisions obtained among all agent-processes of RCGA and agent-particles of PSO, respectively (Fig. 2).

An important feature of RCGA-PSO is the mutation operator applied to each element of the decision-variable vector of a pair of descendant individuals $\widehat{\mathbf{x}}_{1,2,i}(t_k)$, previously obtained as a result of selection and crossing over at the level of each i-th agent-processes $(i \in I)$ at the moment t_k $(t_k \in T)$:

(15) $$\breve{\mathbf{x}}_{1,2,i}(t_k) = \begin{cases} \mathrm{SUM}\left(\widehat{\mathbf{x}}_{1,2,i}(t_k),\ |I|\right) \text{ if } t_k < \frac{|T|}{4} \text{ and } h(0,\ 1) \le 1, \\ \mathrm{MUT}\left(\widehat{\mathbf{x}}_{1,2,i}(t_k),\ u\right) \text{ if } \left(t_k \ge \frac{|T|}{4},\ t_k < \frac{|T|}{2} \text{ and } h(0,\ 1) \le \tilde{p}_m\right) \text{ or} \\ \qquad \left(t_k \ge \frac{|T|}{2} \text{ and } h(0,\ 1) \le \underline{p}_m\right), \\ \widehat{\mathbf{x}}_{1,2,i}(t_k) \text{ if } h(0,\ 1) > p_m. \end{cases}$$

Here: $\tilde{p}_m$, $\underline{p}_m$ are the standard and minimum probability of applying the mutation operator to the previously obtained (through a crossover) $\tilde{\mathbf{x}}_i(t_k)$, $i \in I$, decisions; $\mathrm{SUM}\left(\hat{\mathbf{x}}_{1,2,i}(t_k),\ |I|\right)$ is the scalable uniform mutation operator, the performance of which depends on the total number of involved agent-processes (see [9]); $\mathrm{MUT}\left(\hat{\mathbf{x}}_{1,2,i}(t_k),\ u\right)$ is the combined mutation operator selected from the set of $\{\mathrm{UM, PM, SUM}\}$ (uniform, power, etc.), where $u \in \{1, 2, 3\}$ is the random number defined in accordance with a discrete uniform distribution; $h(0,\ 1)$ is the random number uniformly distributed on the segment $[0,\ 1]$.

3.2. Test results for RCGA-PSO

In Table 1 known test instances that have been carried out to test and verify the developed RCGA-PSO Algorithm are presented.

Table 1. Test instances for RCGA-PSO Algorithm

Test instances	Problem statement and global minimum	Feasible ranges
FT1 – Rastrigin function	$f(\mathbf{x}) = 10n + \sum_{i=1}^{n}\left(x_i^2 - 10\cos(2\pi x_i)\right)$, $f(0, 0, ..., 0) = 0$	$-5.12 \le x_i \le 5.12$, $i = 1, 2, ..., n$
FT2 – Rosenbrock function	$f(\mathbf{x}) = \sum_{i=1}^{n-1}\left(100(x_{i+1} - x_i^2)^2 + (x_i - 1)^2\right)$, $f(1, 1, ..., 1) = 0$	$-5 \le x_i \le 5$, $i = 1, 2, ..., n$
FT3 – Shaffer's F6 function	$f(\mathbf{x}) = 0.5 + \dfrac{\sin^2\left(\sum_{i=1}^{n}\sqrt{x_i^2}\right) - 0.5}{\left(1 + 0.001x_i^2\right)^2}$, $f(0, 0, ..., 0) = 0$	$-100 \le x_i \le 100$, $i = 1, 2, ..., n$
FT4 – Schwefel function	$f(\mathbf{x}) = -\frac{1}{n}\sum_{i=1}^{n} x_i \sin\left(\sqrt{\lvert x_i \rvert}\right)$, $f(420.9685, ..., 420.9685) = -418.98289$	$-500 \le x_i \le 500$, $i = 1, 2, ..., n$
FT5 – Styblinski-Tang function	$f(\mathbf{x}) = \frac{1}{2}\sum_{i=1}^{n}\left(x_i^4 - 16x_i^2 + 5x_i\right)$, $f(-2.903534, ..., -2.903534) = -39.16616n$	$-5 \le x_i \le 5$, $i = 1, 2, ..., n$

Furthermore, the effectiveness of the proposed heuristic algorithm has been compared with other parallel algorithms. An evaluation of performance metrics values has been performed for the following parallel heuristic algorithms:

- RCGA1 is the real-coded genetic algorithm with the SBX crossover operators and mutation based on uniform distribution [21].
- RCGA2 is the real-coded genetic algorithm with the LX crossover operators and PM mutation [22, 23].
- RCGA3 is the multiagent real-coded genetic algorithm that combines various crossover and mutation operators [9, 10].

- PSO is the standard parallel particle swarm algorithm in which an interaction between all agent-particles is carried out to find the global one [15].
- RCGA-PSO is the hybrid real-coded genetic algorithm that combines the multiagent RCGA3 with the PSO.

Optimization experiments have been carried out on a DSWS PRO supercomputer (2×Intel Xeon Silver 4114, 1×NVIDIA QUADRO RTX 6000) with a limited number of iterations $|T| = 100$ and the total number of required variables $n = 10$. The following parameters have been used for the evaluated algorithms:

- the number of process agents in RCGA algorithms is 100;
- the number of particle agents in the PSO and RCGA-PSO algorithms is 100;
- the sizes of the local population (potential solutions of each agent-process) and the parent population in the RCGA and RCGA-PSO algorithms are 100 and 10, respectively;
- the standard and minimum probabilities of applying the mutation operator in RCGA-PSO are $\tilde{p}_m = 0.01,\ \underline{p}_m = 0.001$;
- the frequency of interleaving and exchanging the best potential solutions of RCGA and PSO algorithms in RCGA-PSO is $w = 5$;
- the constants affecting the rate of convergence in PSO algorithms are $\theta = 0.5,\ c_1 = 1.5,\ c_2 = 2.5$.

Table 2. Evaluation of performance metrics of RCGA-PSO

Test instances	Metrics	RCGA-PSO	RCGA and PSO algorithms				Optimum
			RCGA1	RCGA2	RCGA3	PSO	
FT1	$\tilde{F}(\mathbf{x})$	**0.0000**	0.0000	0.0000	0.0000	8.1500	**0**
	$\tilde{\sigma}$	**0.0**	0.0	0.0	0.0	4.4	
	PT	**1.3 s**	20.5 s	20.5 s	21.1 s	0.1 s	
FT2	$\tilde{F}(\mathbf{x})$	**0.0007**	8.9915	8.9932	0.0187	7.3472	**0**
	$\tilde{\sigma}$	**0.0**	0.0	0.0	0.0	2.2	
	PT	**1.4 s**	18.0 s	20.5 s	22.2 s	0.1 s	
FT3	$\tilde{F}(\mathbf{x})$	**0.0000**	0.0000	0.0000	0.0000	0.0380	**0**
	$\tilde{\sigma}$	**0.0**	0.0	0.0	0.0	0.0	
	PT	**1.3 s**	20.4 s	20.6 s	21.1 s	0.1 s	
FT4	$\tilde{F}(\mathbf{x})$	**–402.213**	–351.099	–358.369	–406.375	–374.910	**–418.9829**
	$\tilde{\sigma}$	**1.2**	15.8	17.5	9.9	17.8	
	PT	**1.5 s**	14.9 s	16.8 s	16.1 s	0.1 s	
FT5	$\tilde{F}(\mathbf{x})$	**–391.661**	–298.066	–300.985	–391.655	–388.297	**–391.6612**
	$\tilde{\sigma}$	0.0	10.5	12.3	0.0	6.0	
	PT	1.8 s	22.3 s	22.3 s	22.5 s	0.2 s	

As criteria for the effectiveness of the algorithms under consideration, the following has been used:

- $\tilde{F}(\mathbf{x})$ is the median (among all performed optimization experiments) value of the target functional at the moment $|T| = 100$,
- $\tilde{\sigma}$ is the standard deviation (i.e., the stability of the resulting solutions);

- PT (seconds) is the average execution time of the heuristic algorithm. The test optimization results are presented in Table 2.

It is clear from Table 2 that RCGA-PSO demonstrates the best time-efficiency relative to other Real-Coded Genetic Algorithms (RCGAs), while keeping up the required level of accuracy for obtained decisions. While PSO is characterized by the best time efficiency among all those considered (see Table 2), it is significantly inferior in the quality of decisions (i.e., it is less close to the optimum) in comparison with the most efficient genetic RCGA3 Algorithm. Thus, the hybrid RCGA-PSO has the advantages of both RCGA3 and PSO algorithms in terms of accuracy and optimization rate, respectively.

Fig. 3 shows a graph of the convergence rate of the objective function to the optimum for RCGA-PSO depending on the number of agent-processes (and agent-particles) implementing evolutionary search procedures using the instance of FT1 for $n = 10$.

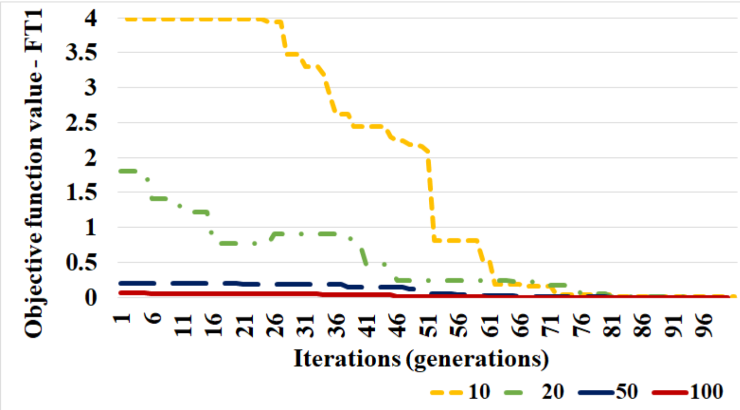

Fig. 3. Convergence rate in RCGA-PSO for FT1

It is evident from Fig. 3 that the convergence rate of RCGA-PSO depends non-linearly on the number of interacting agent-processes of RCGA and agent-particles of PSO. At the same time, the implementation of the substandard algorithm on the FLAME GPU makes it possible to significantly increase the number of such agents compared to the parallelization architecture based on MPI (i.e., Message Passing Interface, see [9]), providing higher convergence rates.

4. Results of simulation and optimization experiments

Fig. 4 shows the results of numerical Monte Carlo experiments performed using the simulation model (1)-(11) with the total number of economic agents $|I| = 2000$ and products $|P| = 10$. The total number of runs performed is 1000. At the same time, the results of Monte Carlo experiments have been used in the construction of the ANN-based surrogate model to provide the fitness-function approximation within RCGA-PSO and reduce the number of recalculation needs to estimate the objective function values.

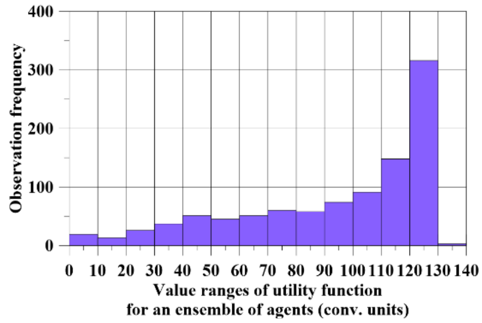

Fig. 4. Frequency diagram of utility function values for an ensemble of economic agents

Table 3 presents the values of the control parameters of the model obtained as a result of the statistical (frequency) analysis of the results presented in Fig. 4.

Table 3. The values of the control parameters obtained with the Monte Carlo Algorithm

Control parameters			Value ranges of utility function (conv. units)		
			0-10	120-129	130-140
Average utility (conv. units)			2.69	85.84	118.75
Parameters defining economic agents' states	μ_b	max	0.5969	0.9997	0.5450
		avg.	–0.7796	0.4744	0.3010
		min	–0.9948	–0.5565	–0.1302
	σ_b^2	max	0.9616	0.9686	0.5578
		avg.	0.2507	0.3368	0.2928
		min	0.0068	0.0014	0.1346
	μ_m	max	0.9806	0.9950	–0.0024
		avg.	–0.0850	0.0298	–0.5025
		min	–0.8478	–0.9798	–0.8587
	σ_m^2	max	0.9682	0.9966	0.5500
		avg.	0.5259	0.5331	0.4001
		min	0.0287	0.0014	0.2780
	Observation frequency of configurations of agents' placement c	1	11	63	3
		2	5	55	0
		3	1	41	0
		4	2	46	0
		5	4	52	0
		6	4	55	0
Environment's characteristics (for assembly of economic agents)	Coefficient of contractuality α	max	0.9865	0.9951	0.3850
		avg.	0.5238	0.5252	0.2569
		min	0.0031	0.0024	0.0866
	Trade interaction radius r	max	20.0	20.0	11.0
		avg.	8.4	11.6	8.0
		min	1.0	3.0	5.0
	Probability of moving agents h	max	0.9865	0.9951	0.3850
		avg.	0.5238	0.5252	0.2569
		min	0.0031	0.0024	0.0866

Table 4 presents the results of optimization experiments obtained using RCGA-PSO for various configurations of the initial distribution of agents in a discrete space, providing the maximization of the average utility of future consumption for an ensemble of agents.

As evident from Tables 3 and 4 increase in the values of the contact coefficient α, trade interaction radius r, as well as the probabilities of moving agents h does not lead to an increase in average utility. At the same time, the choice of the initial configuration for placing agents in a discrete space significantly affects the optimization results. In particular, the first (i.e., mixed) configuration (see Fig. 1) has an advantage, since as a result, the probability of contracts between closely located (initially) agents-sellers and agents-buyers increases. It is noteworthy that with such a favorable configuration, it is advantageous for agents to remain in an almost stationary state, i.e., do not change their position in space $(h = 0.0054)$. The positive impact of the desegregated placement of agents on the socio-economic system has also been previously noted by the authors earlier in [9], in which a multi-sector model of limited neighborhoods has been proposed. In such a model, the increase in contacts between the two groups of agents leads to the acceleration of the processes of assimilation of migrant agents and their subsequent involvement in high-tech sectors of the economy.

Table 4. The optimal values of the control parameters obtained with RCGA-PSO

Control parameters		Configurations of the initial placement of agents in space and optimal values of control parameters					
		1	2	3	4	5	6
		136.54	129.98	131.03	131.12	130.72	131.56
Parameters that define agent states in time	μ_b	0.7383	0.2156	0.4619	0.5121	–0.1939	0.9286
	σ_b^2	0.0268	0.0155	0.2960	0.2120	0.0272	0.0239
	μ_m	–0.6936	–0.1739	–0.8091	–0.5984	–0.7561	0.2702
	σ_m^2	0.2497	0.3254	0.0141	0.0735	0.7010	0.2381
Environment characteristics	Contractuality coefficient α	0.0160	0.1973	0.1193	0.2130	0.1838	0.1391
	Trade interaction radius r	8	8	8	7	7	7
	Probability of moving agents h	0.0054	0.1677	0.0001	0.0004	0.1756	0.0145

Fig. 5 shows the convergence rate of the average utility to the maximum for RCGA-PSO in various configurations of the initial placement of agents in space.

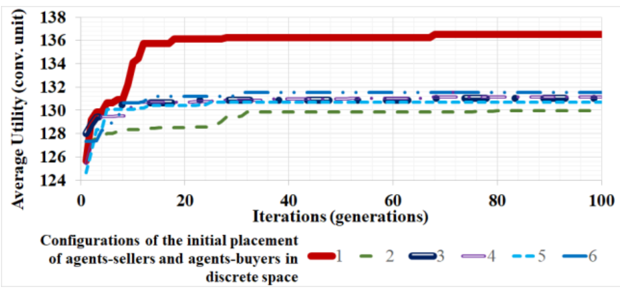

Fig. 5. Dynamics of convergence of the average utility to the maximum

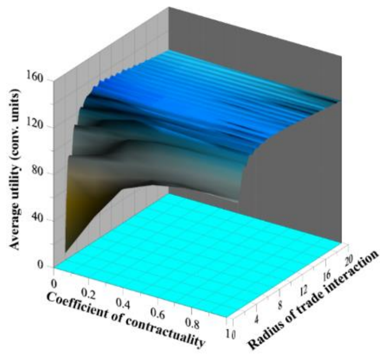

Fig. 6. Dependence of the average utility on values of the most influenced control parameters

In Fig. 6, the dependence of the average utility on the coefficient of contractuality and the radius of trade interaction is presented, obtained for the first (most favorable) configuration of the initial placement of agents, the optimal states of which are formed using RCGA-PSO.

It is clear from Figs. 5 and 6 that RCGA-PSO can be applied to optimize the characteristics of the environment and the strategies for making individual decisions by agents participating in barter and monetary interactions.

5. Conclusion

This paper presents a stochastic agent-based model of the exchange of goods between multiple agents, implemented in the FLAME GPU agent-based modeling environment. The developed model uses the approach, first proposed in [2, 3], based on the study of random barter and monetary interactions to maximize the utility of future consumption of agents making individual decisions about concluding contracts.

For the first time, a hybrid heuristic optimization algorithm is developed that combines the real-coded genetic RCGA Algorithm and PSO. It uses an ANN-based surrogate model to improve the characteristics of the agents' environment and search for optimal strategies for making individual decisions. A new genetic RCGA-PSO Algorithm based on the combined use and interaction of RCGA and PSO has been developed. RCGA-PSO is characterized by the best time efficiency relative to other similar algorithms while keeping up the required level of accuracy of the obtained decisions.

The results of the numerical experiments performed using RCGA-PSO and the developed stochastic model of goods exchange show the particular importance of choosing the initial configuration for agents' placement in a discrete space when solving the problem of maximizing the average utility.

Further research will be directed to the development of new heuristic operators (crossover and mutation), which provide the possibility of more accuracy and autonomous optimization of individual decision-making strategies. Such a system can be applied when analyzing the trade in information goods.

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