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Information Systems Reliability in Traditional Entropy and Novel Hierarchy

Iliyan I. Petrov

Bulgarian Academy of Sciences, Institute for Information and Communication Technologies, Acad. Georgui Bonchev Str., Block 2, 1113 Sofia, Bulgaria
E-mails: petrovindex@gmail.com iliyan.petrov@iict.bas.bg

***Abstract**: The continuous progress of computing technologies increases the need for improved methods and tools for assessing the performance of information systems in terms of reliability, conformance, and quality of service. This paper presents an extension of Information Theory by introducing a novel hierarchy concept as a complement to the traditional entropy approach. The methodology adjustments are applied to a simulative numerical example for assessing the reliability of systems with different complexity and performance behavior.*

***Keywords**: Information theory, entropy, hierarchy, information system reliability*.

1. Introduction and review of previous work

Impressive innovations in computer hardware and software have created new opportunities for information systems together with new challenges for systems administrators and company managers.

Historically, the application of system analysis in modern industries management starts with "Principles of Scientific Management" (T a y l o r [1]), whose influence can be found in several concepts and theories for quality management [2]. In recent decades, several approaches have been developed with the aim to improve the effectiveness of production and information processes, such as the "Total Quality Management (TQM)" [3-6], various international quality standards like ISO 9000-9001 [7, 8], "Six-sigma" [9], "Lean manufacturing" [10, 11].

One of the key aspects in most of the industrial management theories is the relation between the system's quality of performance with the conformity to the users' requirements for optimization of processes and products. Even experienced consultants face challenges when dealing with the problems concerning information flows in the newest computing technologies and sophisticated information systems [12]. Quite often, computer and communication professionals have to rely on abstract judgements and subjective preferences based on information from unreliable sources [13]. In similar situations, the Decision-Makers (DMs) have to make their choices for initial investments and current maintenance under conditions of uncertainty and conflicts of interests between vendors, manufacturers, and other third parties. Since the 1970-s, one of the most popular approaches to exploring the reliability of information systems has been to construct hierarchical configurations with a limited number of Critical Success Factors (CSF) [13-18]. Although pragmatic, such an approach suffers from the lack of uniform listing for CSF and the limited time and computational resources for assessing the specifics in different sectors and organizations [20-22]. This consideration stresses the need for objective and universal models for analyzing the performance of information systems [23].

This paper presents a structured approach that considers the role of all main computing modules and allows for assessing the system complexity by comparing the information related to the performance incidents on the basis of the traditional entropy and novel hierarchy concepts. In this context, the complexity of the system and its reliability is associated with the randomness of resources' and probabilities' distribution among the system's components as a basis for further multi-criteria analysis and decision making. A justification why entropy and hierarchy can be used to assess the complexity and effectiveness of different management and information systems is contained in the basic principles of L. von Bertalanffy [24] and Wiener's concept [26]. Based on statistical measurements, the entropy and hierarchy concepts can provide reliable tools for assessing the status and the performance of information systems [26, 27].

2. Assessing system complexity in entropy and hierarchy

From a methodological point of view, treating of raw data for the relative weights of components and the probabilities of events can be performed with two different concepts: 1) entropy – for measuring directly diversity, competition, uncertainty; 2) concentration (hierarchy) – for measuring directly order, domination and certainty. Traditionally, natural sciences assess the distribution of resources among systems components in terms of diversity, uncertainty, and chaos. Social sciences (economics, competition law, etc.) prefer to consider these issues from an opposite point of view – the concentration of resources reflecting the level of domination and hierarchy in competition interactions, or, in other words, the certainty for the realization of events with different probability distributions. Previous publications [28, 29] have discussed essential methodology aspects and introduced novel tools for assessing system complexity on the basis of novel Hierarchy approaches, comparing them with the most popular existing indicators.

At micro-level (Level-1), the empiric data about individual components parts p_i are treated (filtered) with some non-linear function (called here *basic transform function*) which plays a key role in assessing system complexity. The basic functions conceptualize the specifics of the different approaches on *what* and *how* will be measured: a) disorder and uncertainty in terms of entropy $e(p_i)$; b) order and certainty in terms of concentration/hierarchy $h(p_i)$.

At macro-level (Level-2), the transformed output results for all individual system components are summed to obtain the aggregated result of cumulative Entropy (*E*) or cumulative Hierarchy (*H*):

(1) $$H = \sum_{i=1}^{n} h(p_i),$$

(2) $$E = \sum_{i=1}^{n} e(p_i).$$

2.1. Information theory and traditional entropy

The complexity of systems and their structural evolution are key issues for characterizing the specifics of their dynamics in a large number of areas.

The term “Entropy” was introduced in 1865 by C l a u s i u s [30] as a parameter for describing the internal thermodynamic transformations in isolated systems (“entropy” comes from the Greek word *Entropia* which means transformation).

In the 1870s, B o l t z m a n n [31], G i b b s [32], and M a x w e l l [33] added a statistical dimension to the understanding of entropy by founding the principles of “statistical mechanics”. By 1948, the Clausius concept for measuring entropy $\mathrm{se}(p_i) = -p_i.\log_b p_i$ was borrowed as a key point in the “Information Theory of Claude Shannon” [34] in the format with binomial logarithm basis (i.e., *b*=2):

(3) $$\mathrm{se}(p_i) = -p_i.\log_2 p_i s,$$

(4) $$\mathrm{SE}(p_i) = \sum_{i=1}^{n} \mathrm{se}(p_i) = -\sum_{i=1}^{n} p_i.\log_2 p_i.$$

Fig. 1 displays the graphical visualization of micro-level individual Shannon Entropy $\mathrm{se}(p_i) = -p_i.\log_b p_i$ and the nominal cumulative maximal Shannon Entropy SEmax in the popular of logarithmic bases (*b*=2, *b*=2.718, and *b*=10).

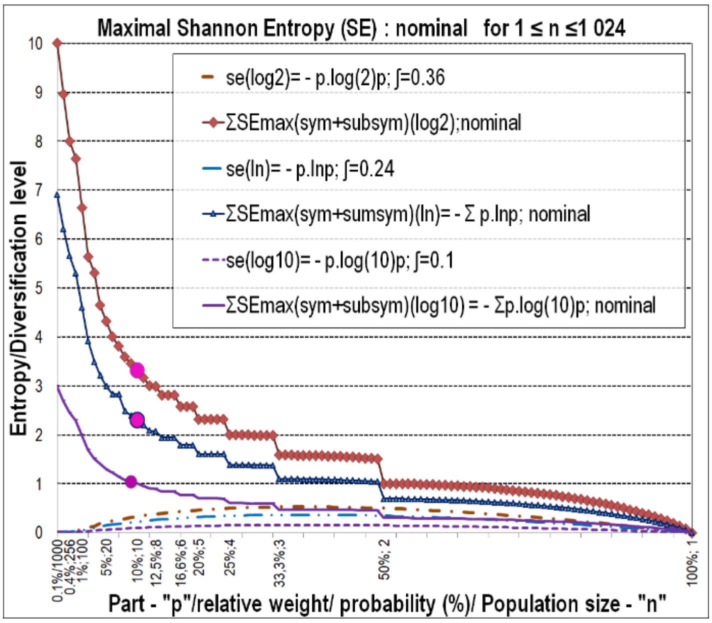

Fig. 1. Individual and nominal Maximal Shannon Entropy function and their integral values in the most popular formats: a) $\log_2$; b) ln; c) $\log_{10}$

The parabolic profile of the Shannon Entropy’s basic function produces a specific ambiguous issue – two very different values of primary data p_i are transformed into one value of individual entropy. For example: in the format with a binomial logarithm basis, both $p_i = 0.25$ and $p_i = 0.5$ result in an equal value of individual entropy: $0.25 \log_2 0.25 = 0.5 \log_2 0.5 = 0.5$. This case is only one example of a large number of such possible pairs that produce the same type of confusion and uncertainty. Also, in the interval $p_i \epsilon(0.2, 0.6)$ the parabola of $\mathrm{se}(p_i)$ possesses a very flattened profile, while at its both ends for $p_i \approx 0$ and $p_i \approx 1$ the individual entropy is practically equal to zero, $\mathrm{se}(p_i) \approx 0$. The mathematical definition of Equation (3) depends only on the variable p_i excluding any possibility for flexible modelling of the basic and cumulative function. In addition, the necessity to perform the common procedure of normalization may also create undesired confusion since higher maximal levels of entropy used for normalization produce more unbalanced results. From this point of view, the ambiguous ranking of individual entropy values creates uncertainty in using Shannon Entropy as a universal method for classification.

2.2. Logistic Hierarchy (LH): consistency, balance, flexibility

The main priority for modeling novel indicators in our approach is to limit the possible "distortion effect" and the loss of primary information in the process of transforming empiric input data about parts p_i into output individual hierarchy values. At the same time, the efforts are focused on obtaining a balanced distribution of cumulative system information at the macro-level by selecting a value for the power constant *b* which will position the value for minimum cumulative hierarchy for a fully symmetric macro-state with 10 components with equal "parts" $p_1 = p_2 = \cdots = p_{10} = 0.1 = 10\%$ at the central level of 0.5.

The basic transform functions can be modeled with different mathematical models. One of them is the so-called *logistic function concept*. It defines functions with a specific *S*-type (sigmoid) graphical profiles that monotonically increase with the increase of the values of the basic variable parameter, which in our case is p_i (relative weight or event probability) of an individual component. Logically, such monotonically increasing *S*-profiles are particularly suitable for assessing the system complexity and representing the structural evolution of information from the point of view of domination and certainty. A higher value of p_i logically reflects stronger domination and more information in terms of order and competitive strength.

To enlarge and improve the analysis of system complexity an original concept called *logistic hierarchy* (lh) is introduced and defined as follows:

(5) $$\mathrm{lh}(\log) = \frac{p_i}{1+\left(\frac{\sum_{j=1}^{J} \log_{R_j} p_i}{J}\right)^{c}},$$

where: p_i are the part/relative weight/probability of micro-states; $p_i \epsilon\ [0, 1]$;

n is the number of components in the system populations;

R_j are the referral weights or Reference Structural Thresholds (RST); $R_j \epsilon [0, 1]$;

J is the number of Reference Structural Thresholds (RST) or R_j;

c is the the intensity of competition interactions; $c \epsilon [1, 3]$.

The key role in the novel basic function concept lh belongs to the module $\left(\frac{\sum_{j=1}^{J} \log_{R_j} p_i}{J}\right)$ which in the format of arithmetic average accumulates and generalizes the information about the possible "logarithmic comparisons" of individual components weights p_i with a flexibly definable set of reference values RST R_j used as respective logarithm bases. As a result, the logistic hierarchy lh approach allows modelling flexibly an unlimited number of functions with different *S*-curve profiles. In all of them, primary data p_i is non-linearly transformed in the universal dimensionless scale interval [0, 1], without any ambiguity on the micro-level or a need for normalization on the macro-level. Several simulations have been conducted to select the variant with a set with two RSTs ($j = 2;\ R_1 = 0.001; R_2 = 0.25$) and c=2 for defining the basic function lh(log:0.001;0.25) as follows:

(6) $$\text{lh}(\log: 0.001; 0.25) = \frac{p_i}{1+\left(\frac{\log_{0.001} p_i+\log_{0.25} p_i}{2}\right)^2}.$$

Fig. 2 includes the graphics of the following functions:

a) basic transform function in the variant lh(log:0.001;0.25);

b) cumulative Minimal Hierarchy in the “Equalization in each population” scenario, formed by the sets of points for symmetric and sub-symmetric system configurations;

c) cumulative Maximal Working Hierarchy in the “Single leader domination decreasing in all populations” scenario, formed by system configurations with maximal asymmetric distribution of p_i between the leaded and its competitors;

d) cumulative Mean Working Hierarchy, formed by the arithmetic mean of “minimal and sub-minimal hierarchies” and the “maximal working hierarchy”.

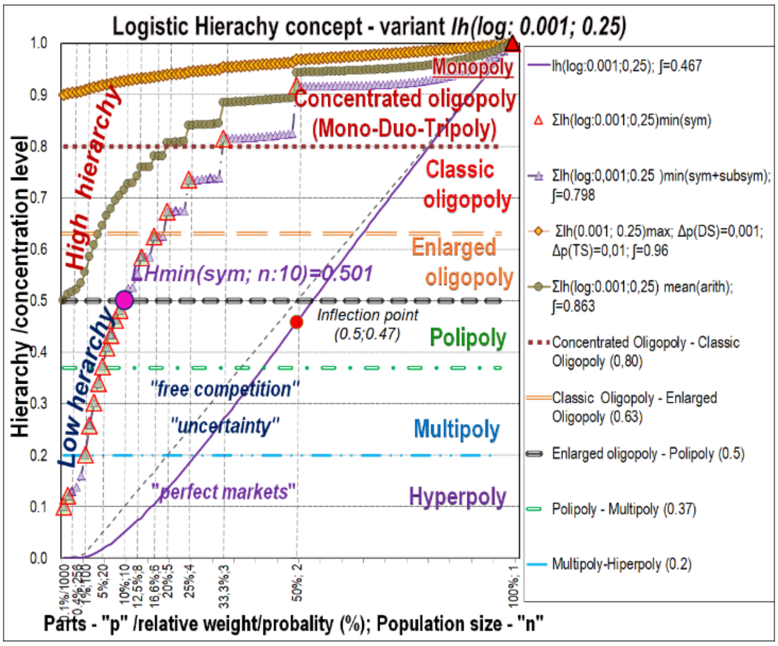

Fig. 2. Modeling of individual and cumulative Logistic Hierarchy functions (Minimal, Maximal and Mean Working levels): variant lh(log:0.001;0.25)

2.3. Power function model: novel adjustment for Hierarchy

As a next step, it is possible to define a model with a profile similar to lh(log:0.001;0.25) in the popular traditional “power function model”:

(7) $$f(p_i) = a(p_i)^b + c.$$

A very simplified variant of this popular power function concept with a=1, b=2, and c=0 is used in the basic function of the Herfindahl Concentration defined as $\mathrm{hc}(p_i) = p_i^2$ which has the disadvantage of producing very unbalanced results at micro and macro-level [35, 36]. To make the model simpler and similar to the format of Herfindahl Concentration the experiments have shown that such approximation can be achieved with a=1, b=1.3, and c=0:

(8) $$\mathrm{ph}(\mathrm{pf} = 1.3) = (p_i)^{1.3}.$$

Fig. 3 includes the graphics of the following functions:

a) basic transform function in the variant ph(pf=1.3);

b) cumulative Minimal Hierarchy in the "Equalization in each population" scenario, formed by the sets of points for symmetric and sub-symmetric system configurations;

c) cumulative Maximal Working Hierarchy in the "Single leader domination decreasing in all populations" scenario, formed by system configurations with maximal asymmetric distribution of p_i between the leaded and its competitors;

d) cumulative Mean Working Hierarchy, formed by the arithmetic mean of "minimal and sub-minimal hierarchies" and the "maximal working hierarchy".

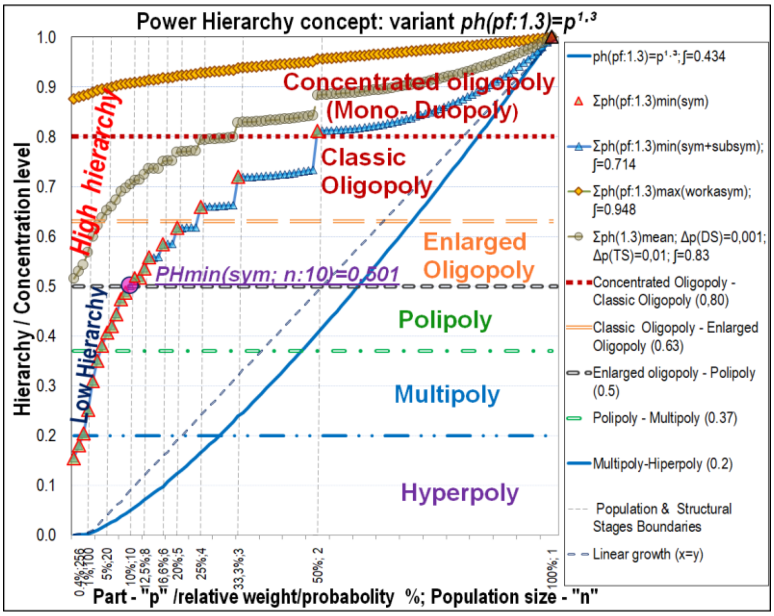

Fig. 3. Modeling of individual and cumulative Power Hierarchy functions (Minimal, Maximal and Mean Working levels): variant ph(pf=1.3)

To our knowledge, clear and balanced rules for universal framing of information space and classifying system complexity have not yet been defined in the context of Shannon Entropy. The integral values and the graphical profiles of the cumulative minimal and maximal function for lh(log:0.001; 0.25) and ph(pf=1.3) are similar and fit closely in the "Harrington scale", which in our more detailed variant contains 6 intervals and represents an adapted version of classical *E. Harrington's preference function* for psycho-physical classification [37], as shown in Table 1.

Table 1. Harrington scale: adaptation with six quantitative intervals and six hierarchy phase labels

Hierarchy phases labels	Harrington scale (quantitative values)	Traditional qualitative labels
Concentrated oligopoly	0.80-1.00	Extremely high
Classic oligopoly	0.63-0.80	Very high
Enlarged oligopoly	0,50-0,63	High
Polipoly	0.37-0.50	Low
Multipoly	0.20-0.37	Very low
Hyperpoly	0.00-0.20	Extremely low

The central value of 0.5 in the Harrington classification scale is meant to correspond to the official competition law regulations in the EU, Japan, Russia, and other countries (incl. USA until 2010) for a fully symmetrical market with 10 competitors when all of them have equal shares of $p_i = 0.1 = 10\%$ [38, 39].

Fig. 4 displays the Level-2 aggregation for "symmetric + sub-symmetric" configurations, which define the cumulative "maximum + sub-maximum" levels of entropy (normalized Shannon Entropy in the three most popular logarithmic formats ($\log_2 p_i$, $\ln p_i$, $\log_{10} p_i$), and the "minimum + sub-minimum" levels of hierarchy for three different concepts: the traditional Herfindahl Concentration (HC) indicator and the novel hierarchy indicators in the variants lh(log:0.001;0.25) and ph(pf=1.3).

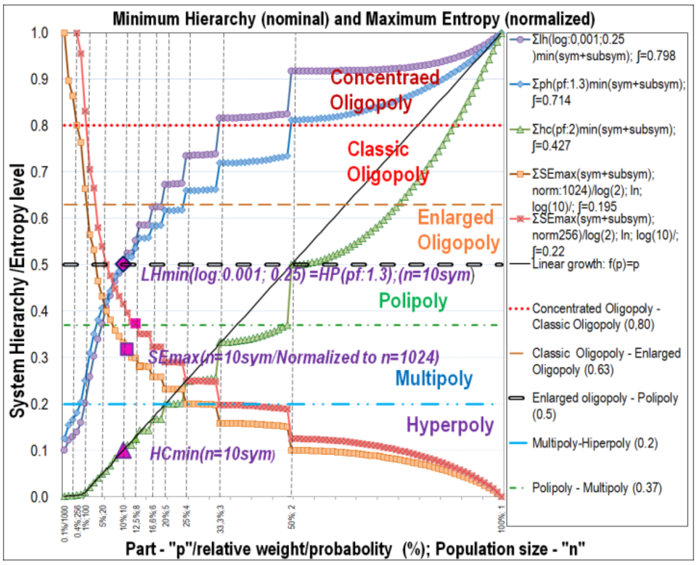

Fig. 4. System complexity Indicators: Minimal Hierarchy and Maximal Entropy

At the micro-level, the transformation of $p_i = 0.1$ results in lh(log:0.001; 0.25)≈0.0501, the minimal cumulative Logistic Hierarchy for the system with 10 equal components (all equal to $p_i = 0.1$) is $lh(\log:0.001;0.25) =$ $(\sum_{i=1}^{n=10}$ 0.0501=10×.0501≈0,5. At the same time, the minimal cumulative hierarchy for the same type of symmetric configuration with the same 10 equal components in the case of the Power Hierarchy concept in the variant $\mathrm{ph}(\mathrm{pf} = 1.3)$ is: $\sum_{i=1}^{n=10}(0.1)^{1.3}$=10×0.0501≈0,50. At the same time, the cumulative Minimal Herfindahl Concentration produces a very unbalanced profile – the cumulative concentration for the symmetric configuration with 10 equal components is far from the central point of 0.5: $\mathrm{HC(pf=2)}=\sum_{i=1}^{n=10}(0.1)^2$=10×0.01=0.1. In the case of the Shannon Entropy, such comparisons with other methods are possible only after normalization with a higher level of maximal cumulative entropy. The normalization is made with a maximal cumulative entropy for a system with 1024 components (n=1024) is $\mathrm{SEmax}(n=1024)=-\sum_{i=1}^{n=1024}\frac{1}{1024}\log_2\frac{1}{1024}\approx 10$. Consequently, the normalized maximal cumulative entropy value for a system with 10 equal components is:

$$\mathrm{SEmax}(n{=}10;\ \mathrm{norm}{:}\ n{=}1024)=\frac{1}{10}\left(-\sum_{i=1}^{n=10}\frac{1}{10}\log_2\frac{1}{10}\right)=0.332.$$

Unfortunately, the result of such normalization will be different for any different value of maximal entropy selected as a reference for normalization. For example, in the case of normalization with a maximal cumulative entropy for a system with 256 components (n=256) the maximal cumulative value for a system with 10 equal components is: $\mathrm{SEmax}(n{=}10;\ \mathrm{norm}{:}\ n{=}256)=0.37$. These values are also far away from the central point 0.5 in the Harrington classification scale, and, therefore, cannot be considered as an intuitive and balanced border between high and low cumulative entropy.

The logistic hierarchy in the variant lh(log:0.001; 0.25) has been successfully applied to assess system complexity, market evolution, and competition interactions in the energy sector since 2015 [40, 41].

3. Case study: assessment of information systems reliability in entropy and hierarchy

Under conditions of economic and technical constraints, the risk analysis of information systems performance allows for optimizing the allocation of resources and improving the reliability of their functioning [42, 43]. Entropy-based measures have been used successfully in computing technology – predominantly for cryptography, software reliability, and other areas [44, 45]. Logically, entropy is relatively low in systems characterized by high reliability and high in systems with low reliability and conformance. On the opposite, higher levels of hierarchy characterize systems with high reliability, while low levels – systems with low reliability and conformance. An important preliminary condition for performing effective entropy and hierarchy assessment and analysis is the usage of reliable empiric statistical data which has to be systematically collected and well structured. In this respect, a logical and thorough list of system components represents the first step in assessing the reliability and conformity of information systems.

In the analysis of information systems, users' requirements can be linked to respective system components with two possible realizations – positive (order or conformance) and negative (defect or non-conformance). In this context, the functioning of information systems can be regarded as a Markov process in which the realization of subsequent probabilities depends on the realization of previous events. In other words, the present and future distribution of resources or probabilities among individual components are related to previous micro-states and distributions.

The building of understandable and reliable statistics for any system traditionally starts with framing the procedures for collecting and treating the raw data. In this case, the number of all negative events n (defects or non-conformance incidents) in an information system is distributed among a finite number of components. To simplify the calculations, all individual events (incidents) are attributed equal importance – in other words, equal relative weights or probabilities $(p_i = 1/n)$.

Since the process is of Markov type, the sum of all discrete probabilities about individual incidents or respective components is equal to 1 (100%). In such an egalitarian framework and in terms of entropy, the most sensitive and risky component of the system performance is related to the highest number of individual incidents. If all components are out of conformance with an equal number of incidents, the level of entropy (disorder) in the system is at the highest level for this number of components. In other words, the maximal uniform distributions of individual incidents among components result in maximum uncertainty and this reflects "the principle of maximum entropy" [46, 47]. Naturally, if all incidents are linked to only one component the probability of its defectiveness (non-conformance) is equal to 1 and the cumulative Shannon Entropy is zero. This "information paradox" is part of specific entropy logic in which "more entropy" means "less information".

In the context of hierarchy and concentration, the most sensitive and risky component in system performance is also related to the highest number of individual incidents. If all components are out of order by an equal number of incidents, the hierarchy and order in the system are at the lowest level for that number of components. In other words, the maximal uniform distribution of incidents results in minimum certainty and reliability. If all incidents are linked with only one component, the hierarchy (certainty) of defects is maximal and equal to 1. The cumulative system hierarchy is 0 only when all events probabilities are 0. Naturally, this kind of logic is more intuitive, straightforward, and non-ambiguous – "more hierarchy" means more information, more order, and more certainty.

Such arrangements are useful in two main aspects about the information in the system:

1) The increase in the value of cumulative entropy reflects an increasing disorder in the system; on the opposite – the increase in the value of hierarchy indicates the increase of order and certainty in the system.

2) The values of individual entropy and hierarchy can be used to estimate and adjust eventual Intervention Costs (IC) for each defective component. This can improve the quality of analysis and provide more objective criteria for the prioritization of alternatives optimization of the decision process under conditions of uncertainty and different resource constraints. Such an approach adds valuable economic and efficiency dimensions to the common technical assessment.

To compare the effectiveness of the traditional entropy and the novel hierarchy, this paper presents a case study with two different information systems. For performing the assessments in a more structured way this research uses a list of system components that do not have the pretension to be exhaustive but still tries to consider the main items that could be affected by some kind of reliability incident.

A typical approach for classification in information systems contains usually 3 main groups of components – computing, peripherals, and networks. This traditional triad of technical subsystems may be enlarged with two important elements: the passive element “data” and the active element “staff”. To collect reliable statistics, the frequency of negative events for reliability (defects) can be recorded as “incidents” for some reasonable periods of time (week, month, etc.). In real conditions, the length of this period will depend on the system characteristics and the needs of the users. Obviously, if during the observation period there is no incident, then all components and the system as a whole are in optimal operational condition (“zero defect” status). Situations without incidents are very rare and therefore, this research explores more realistic cases – with different types and number of incidents.

Table 2 contains the comparative results for the cases of User A and User B.

Table 2. Comparative analysis of information systems reliability in entropy and hierarchy

Users	User A					User B				
Components	Incidents	p_i	se(log2)	lh(p_i)	ph(p_i)	Incidents	p_i	se(log2)	lh(p_i)	ph(p_i)
Computing										
CPU										
PSU						1	0.02	0.11	0.005	0.006
RAM						3	0.06	0.24	0.024	0.026
HDD	4	0.5	0.5	0.46	0.41	7	0.14	0.40	0.080	0.078
GPU						1	0.02	0.11	0.005	0.006
OS	1	0.125	0.375	0.07	0.07	5	0.10	0.33	0.050	0.050
Software						2	0.04	0.19	0.013	0.015
Firmware						1	0.02	0.11	0.005	0.006
Staff						1	0.02	0.11	0.005	0.006
Subtotal	*5*	*0.625*	*0.875*	*0.53*	*0.48*	*21*	*0.42*	*1.60*	*0.192*	*0.194*
Peripherals										
Printer	1	0.125	0.375	0.07	0.07	9	0.18	0.45	0.110	0.108
Staff						1	0.02	0.11	0.005	0.006
Subtotal	*1*	*0.125*	*0.375*	*0.07*	*0.07*	*10*	*0.2*	*0.56*	*0.115*	*0.114*
Network										
Switch	2	0.25	0.5	0.18	0.16	10	0.20	0.46	0.130	0.128
Routers						4	0.08	0.29	0.060	0.048
Cables						1	0.02	0.11	0.005	0.006
WiFi						1	0.02	0.11	0.005	0.006
Firmware										
Staff						1	0.02	0.11	0.005	0.006
Subtotal	*2*	*0.25*	*0.5*	*0.18*	*0.16*	*19*	*0.38*	*1.08*	*0.205*	*0.194*
Total	8	1	1.75	0.78	0.72	52	1	3.24	0.512	0.502

For simplification purposes in this paper, the occurrences of all types of incidents are attributed with the same value of importance. In other words, all single incidents n have equal weights (p_i=1/n). Consequently, the importance of each component in the reliability analysis is defined by the number of its incidents. Also, the information systems of the two users have identical configurations from a technical point of view – in other words, they contain the same type and number of components.

The case of **User A** refers to a recently installed information system that faces a limited number of incidents with four exceptions related to the following components: a) data storage on HDD – four incidents; b) Operating System (OS) – one incident; c) printer/scanner overloading – one incident; d) an insufficient number of Switch ports – two incidents. Total of incidents is eight. The cumulative nominal Shannon Entropy is $\sum_{i=1}^{n=8} \text{se}\left(\frac{1}{8}\right) = 8 \times 0.21875$ =1.75. However, without normalization, it is difficult to consider "how low" or "how high" this nominal value is. The other disadvantage discussed above in Section 2.1 (Traditional Entropy and Information theory) is that for very different values of primary data about weights (probabilities) the basic function $\text{se}(p_i)$ produces the same transformed values of individual entropy. Actually, for HHD – p_i= 0.5 results in se(0.5)=0.50 as well as for Switch – p_i=0.25 results in se(0.25)=0.50. Such "ambiguous pairs" are produced by the non-monotonic convex parabola profile of the basic transform function that creates uncertainty for using the Shannon Entropy as a stand-alone method. In these situations, further analysis would require to return to primary data or to use other measuring tools, such as "dispersion" or "variance" which, however, have another peculiarity – for all distributions with equal components they produce undistinguishable "zero results". In the novel concepts for logistic hierarchy lh(log:0.001;0.25) and power hierarchy ph(pf=1.3) the transformation and normalization issues are treated consistently and effectively thanks to their monotonically increasing basic functions. On the micro-level, all transformed output results are ranked in strict accordance with the ranks of their respective input values for the relative weights or probabilities event (p_i).

For the system of User A, the value of the cumulative system hierarchy lh(log:0.001;0.25) is 0.78 and for ph(pf=1.3) is 0.72. These values are different but still very similar, and considerably higher than the central value of 0.5 on the universal scale (0, 1). In the Harrington classification scale with a set of 6 quantitative intervals and, respectively, 6 qualitative phase labels (Table 1, above) both values belong to the phase of *Classic oligopoly* (0.63-0.80). Such a level of cumulative hierarchy indicates that urgent interventions are needed for some limited number of components in the system – the priorities are clearly identified and ranked by the different values of individual hierarchy:

- for HHD: p_i=0.5 results in lh(0.5)=0.46 and ph(0.5)=0.41;
- for Switch: $p_i = 0.25$ results in lh(0.25)=0.18 and ph(0.25)=0.16;
- for OS and Printer/Scanner: $p_i = 0.125$ results in lh and ph in a very similar value of approximately 0.07.

Naturally, in situations when two or more components register equal numbers of incidents which result in equal defect probabilities and equal individual hierarchy values the analysis and prioritization will need further investigation. It can be refined by taking into account the respective "Intervention Costs" and "Impact Damages" for each of the items. This paper presents only the first step – a non-linear entropy and hierarchy assessment for obtaining a holistic view of the system and the influence of individual components on the reliability of IS. It does not take into account the type and the level of priorities, and the costs related to them. For example, a defective HDD causes, in general, the same type of problem in servers, storage equipment, and a home system. However, the role and the quality of these components are very different and their costs will vary substantially. The priority and cost issues are very important and will be addressed in future publications.

The case of **User B** is more complicated: 50 incidents are distributed unevenly between 16 affected components and across the three sub-systems. The cumulative Shannon Entropy is calculated $\sum_{i=1}^{n=16} \mathrm{se}(p_i) = 3.24$. This value is 1.85 times higher than the result in User A (1.75), and without normalization cannot fit in the universal interval (0, 1). In the context of hierarchy, the cumulative results for the novel indicators are very similar: lh(log:0.001;0.25)=0.512 and ph(pf=1.3)=0.502. Compared to User A they reflect a substantial decrease in terms of order or increase of disorder, respectively, 27 and 21 "basic points". In the novel hierarchy approaches, the structural evolution seems much smaller than in the format of Shannon Entropy. However, here the assessment is produced directly on the universal scale of (0, 1) – in other words, the structural space is denser and contains more information per unit.

From a quantitative point of view, the cumulative value of lh(log:0.001:0.25)=0.512 and ph(pf=1.3)=0.502 are insignificantly higher than the central value 0.5. From a qualitative point of view, according to the Harrington scale (Table 1), these values are slightly higher than the central point of 0.5 and the boundary between the phases of *Enlarged Oligopoly* and *Polipoly*. In other words – at the formal border between "high" and "low" levels of hierarchy, and more general – between "order/certainty" and "disorder/uncertainty". These values indicate that the system of User B is in a more vulnerable status. Therefore, User B should undertake more active and urgent measures in several directions: inspecting, replacing or upgrading hardware, updating soft and firmware, inspecting network equipment and cabling, reviewing information flows, and considering sensitive staff issues.

4. Limitations, conclusions, future work

This article presents a comparison application for assessing the reliability of information systems with the traditional indicator for entropy and the novel indicators for hierarchy. Even the limited volume of this publication and simplified case study with only two different numerical examples allows for comparing the efficiency of traditional and new approaches.

In particular, the traditional Shannon Entropy concept contains an inherent ambiguity when assessing small and big components or probabilities, and therefore is suitable for assessing systems in which the distributions include components with individual weights and probabilities lower than the value $p_i = 0.37$ at which the basic transform functions for entropy reach their maximums in the format with any logarithm bases.

The complicated issues about the ambiguity of individual entropy of components and the normalization of cumulative entropy of the system can be mitigated with the proposed hierarchy concepts. The variants of logistic hierarchy lh(log:0.001;0.25) and power hierarchy ph(pf=1.3) produce comprehensive models including basic and cumulative functions with monotonically increasing profiles that ensure fewer information losses and better-balanced profiles with very similar integral values. These findings allow for enlarging the methodology and improving the accuracy of analysis of information systems' complexity and reliability. The novel hierarchy indicators can find application in a large number of areas, where dynamic systems are in constant evolution concerning the access, distribution, and competition for material and non-material resources, such as computer science and technologies, health care and biology, industry (energy, computer, and communication, food, etc.), services (finance, banking, insurance, tourism, trade, e-Trade), leisure and sports, science (innovations, artificial intelligence).

Future studies can be oriented on more detailed reliability and conformity analysis taking into account very important issues like intervention costs and priority of incidents in different information systems. In this respect, it is feasible to improve the hierarchy approaches by introducing additional methods and tools for considering the specific needs of the users and their information systems in the area of multi-criteria analysis and decision support systems.

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