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Development of a Scheme for Correcting Arbitrary Errors and Averaging Noise in Quantum Computing

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***Abstract*: *Intensive research is currently being carried out to develop and create quantum computers and their software. This work is devoted to study of the influence of the environment on the quantum system of qubits. Quantum error correction is a set of methods for protecting quantum information and quantum state from unwanted interactions of the environment (decoherence) and other forms and types of noise. The article discusses the solution to the problem of research and development of corrective codes for rectifying several types of quantum errors that occur during computational processes in quantum algorithms and models of quantum computing devices. The aim of the work is to study existing methods for correcting various types of quantum errors and to create a corrective code for quantum error rectification. The scientific novelty is expressed in the exclusion of one of the shortcomings of the quantum computing process.*

***Keywords*: *Quantum register, quantum computer simulator, complex plane, qubit, quantum error, phase amplitude.*

1. Introduction

Within this paper the basic types of quantum noises are presented. Their fundamental features and their influence on description of density matrices with a certain size and a set of functions are considered. The field of quantum error correction is poorly studied. One of the promising works in this area is [1]. It considers a method-protocol of quantum noise correlation similar to that described in the paper in terms of its basis. It allows visualizing the correlations between pairs of qubits, which helps in detecting two-qubit correlations. The method proposed in this paper eliminates these limitations and allows one to find and eliminate certain types of errors in n-qubit correlations on any number of qubits. An increase of uncorrected data value as a result of the simulation process in accordance with the size of quantum circuit and number of quantum bits [2] is determined. A direct correlation of known types of quantum noise effects on quantum gates and on the result of quantum algorithms [3] is reflected. Successful development of quantum information technologies is impossible without achievement of accuracy of quantum calculations. This requires solving the problem of execution of quantum gates [4] and wired circuits in interaction with various variations of quantum noise types. Next aim – carrying out subsequent analysis of accuracy of performed quantum computing process.

2. Types of quantum noise

Depolarizing noise. A quantum depolarizing channel is a model for quantum noise in quantum systems. The *d*-dimensional depolarizing channel can be viewed as a completely positive trace-preserving map Δ_λ, depending on one parameter λ. This parameter maps a state p onto a linear combination of itself and the maximally mixed state and *d* – number of qubits, *I* is a unit matrix

(1) $$\Delta_\lambda(p) = (1-\lambda)p + \frac{\lambda}{d} I .$$

The condition of complete positivity requires λ satisfying the bounds

(2) $$0 \le \lambda \le 1 + \frac{1}{d^2 - 1} .$$

This type of noise has an effect on the density matrix *p* of dimension *s*s*:

(3) $$p \to \frac{pI}{s} + (1-p) \times U \times p \times U' ,$$

where *I* is a unit matrix. With some probability *p* the base state is replaced with an absolutely random one, and with probability 1–*p* a unitary transformation is performed using the matrix *U*.

Amplitude and phase relaxations. A relaxations usually means the return of a perturbed system into equilibrium. Each relaxation process can be categorized by a relaxation time *τ*. The simplest theoretical description of relaxation as function of time t is an exponential law exp (–*t*/*τ*) (exponential decay). Krauss operators in terms of dephasing are represented as the matrices:

(4) $$E_0 = \begin{Vmatrix} 1 & 0 \\ 0 & \sqrt{1-\gamma} \end{Vmatrix}, \qquad E_0 = \begin{Vmatrix} 0 & 0 \\ 0 & \sqrt{1-\gamma} \end{Vmatrix}.$$

The density matrix applied has the form

(5) $$p_{\text{in}} = \begin{Vmatrix} a & b \\ b^* & 1-a \end{Vmatrix},$$

where *a* is an arbitrary real value, *b* is an arbitrary complex value under this transformation will be equal to 1. The procedure for dephasing quantum states has ben first studied in spin-spin relaxation of spins of several nuclei. Next, the amplitude relaxation procedure is performed. Krauss operators are represented in the form:

(6) $$E_0 = \begin{Vmatrix} 1 & 0 \\ 0 & \sqrt{1-\gamma} \end{Vmatrix}, \qquad E_0 = \begin{Vmatrix} 0 & \sqrt{\gamma} \\ 0 & 0 \end{Vmatrix}.$$

The index *γ* characterizes the relaxation probability, which is related to the transition from $|1\rangle$ to $|0\rangle$. Probabilistic relaxation is an iterative and parallel process, which can be regarded as a recurrent dynamical system. A dynamic system is a set of elements for which a functional relationship between time and the position in the phase space of each element of the system is specified. This mathematical abstraction allows you to study and describe the evolution of systems in time. The state of a dynamical system at any moment of time is described by a set of real numbers (or vectors) corresponding to a certain point in the state space. The evolution of a dynamic system is determined by a deterministic function. The system will assume a specific state, depending on the current one.

3. Averaging noise to reduce errors

We can say that ECC (Error Code Corrections) [5] works by "noise averaging". Since each bit of data affects many transmitted symbols, noise-distorting some symbols usually allows the original user data to be extracted from other. Interleaving ECC-encoded data can reduce the all-or-nothing properties of transmitted ECC codes when channel errors tend to occur in packets. However, this method has limitations; it is best used for narrowband data. Most telecommunications systems use a fixed channel code designed to withstand the worst-case expected bit error rate and then fail at all if the bit error rate gets worse.

Let's define a machine *B* that divides the entire frequency of the visible spectrum by $F_1, F_2, ..., F_M$ and assigns an array of qubits (a set of spatially ordered states [6]) *Q* to each subset of *F*. *B* must act as a bijective function between the frequency partition of the subset *F* and the set of states of the qubit *Q*. It is not allowed to have the same distribution of qubit states for two different frequency subsets. Suppose machine *B* only produces qubit states $|0\rangle$ and $|1\rangle$. The number of colours that can be represented in an array of *n* qubits -2^n as in the classical case, but if turn on the states of the qubits $|-\rangle$ and $|+\rangle$. Then the number of different colours that can be stored – 2^{In}. Formally speaking, the previously observed measurement of control qubits $|p\rangle$ is $A^{\sim} = \alpha_1 |0\rangle\langle 0| + \beta_1 |1\rangle\langle 1|$, α_1, β_1 is the probability of obtaining a quantum states. However, as a solution to this problem were Quantum Error Correcting Codes (QECC) have been proposed. The main difference between QECC and classic codes is to correct the error "without delving into the meaning" of a quantum state. The interaction of a qubit with the environment can lead to an error of one of three possible types:

1. Bit (or, as they are also called, amplitude) errors – *X*, i.e., leading to a qubit flip: $|0\rangle \rightarrow |1\rangle$, $|1\rangle \rightarrow |0\rangle$.
2. Phase errors – *Z*: $|0\rangle \rightarrow |0\rangle$, $|1> \rightarrow -|1\rangle$.
3. Bit-phase (amplitude-phase) errors (i.e., simultaneous action of errors of the first and second types) – *Y*: $|0\rangle \rightarrow |1\rangle$, $|1\rangle \rightarrow -|0\rangle$.

4. Simulation of quantum algorithms implementation

We perform a simulation of the noisy quantum Fourier transform and the quantum Grover algorithm. The main difficulty of such modeling is the exponential growth of the dimension depending on the number of qubits. So, to simulate only 50 qubits, you need to work with vectors from 2^{50} complex numbers. When using single precision numbers, a minimum of 1,125,899,906,842,624 bytes is required to store such a vector. On modern personal computers, using random access memory, approximately 30 qubits can be simulated. So, the simulation of circuits with 32 qubits with 16 Gb of RAM has been implemented. The use of supercomputers makes it possible to increase the number of simulated qubits up to about 40. The quantum Fourier transform is a variation of the discrete Fourier transform, which plays a role in quantum computing simulation algorithms. On initial states, the transform acts as formal model of the proposal: $|j\rangle \to \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} \exp\left(\frac{i \times 2\pi \times j \times k}{N}\right) |k\rangle$, where N is the number of qubits, i, j are two quantum states at the beginning and end of the computational process, k is number of gates on a quantum circuit. A simulation of ideal and noisy quantum algorithms has been performed. Certain types of noise have been chosen for the gates. The Hadamard gate with an admixture of noisiness:

$$H_{\mathrm{e}} = H \times U(\theta), \quad U(\theta) = \begin{Vmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{Vmatrix}, \quad \theta = e \times \xi, \quad \xi \cong N(0,1). \tag{7}$$

Here e is the error level, $N(0, 1)$ is a normal random variable with zero mean and unit variance, θ is the angle of the spin direction in the three-dimensional space of the Bloch sphere. ξ is a set of instructions for undulating quantum errors. All experiments have been carried out on a developed software model of a quantum computing device with a set of specialized libraries and functions. The model is a desktop program written in Java with the help of libraries (API libquantum and LinerAl) of quantum primitives and functions. The general interface of the developed model [7] is shown in Fig. 1. On the left are the buttons for controlling the quantum circuit. This area of the program gives the possibility of automatic or step-by-step movement of the model forward or in the opposite direction. User can also delete the last element selected and entered into the diagram, or completely clear the entire diagram. At the top, there is a menu bar for managing and configuring the model. In the middle at the top is a set of quantum gates, at the bottom is a state diagram of x-registers and y-registers. As a formal model of the proposed quantum computing system having been developed lies in the software interpretation of the fundamentals of quantum computing with the help of quantum software libraries. The library has a set of gates in the API for working with quantum registers. A distinctive feature of this library is the method of storing the state matrix of the qubit group. The memory contains only those states whose probabilities are not equal to zero; this feature allows saving a large amount of memory.

Conclusions from numerical experiments, which are necessary to assess the effect of quantum noise on the accuracy of execution and results of the quantum Fourier algorithm, are shown in Fig. 2. Let us take the probability of coincidence F between ideal and noisy state vectors. This parameter is calculated as the square of the modulus of the scalar product of the corresponding vectors.

If the number of qubits is from 5 to 30, you have to run the algorithm for 300 runs. If the number of qubits is 29, there are 143 runs, and for 33 qubits, there are 26 runs. The curves represent a Gaussian approximation (Fig. 3).

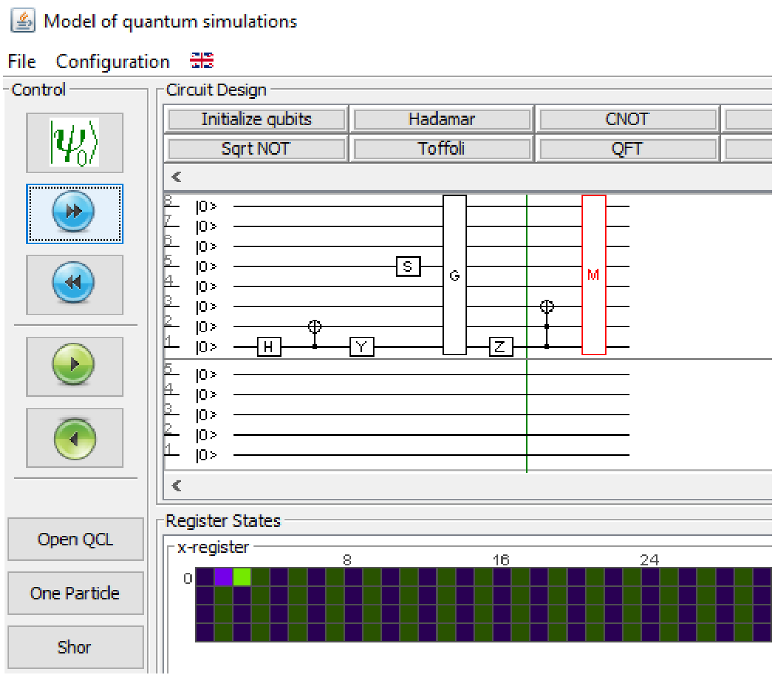

Fig. 1. The interface of the developed modeling environment

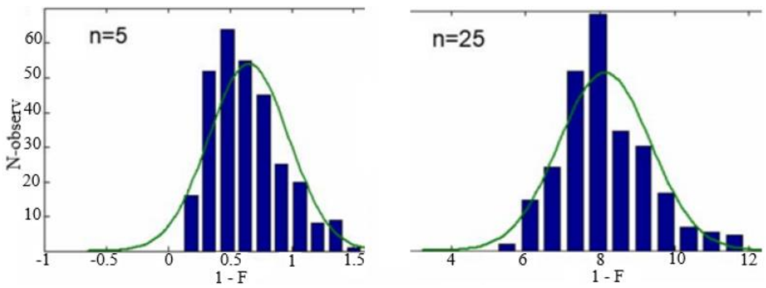

Fig. 2. Cumulative accuracy loss for quantum algorithm from a set of qubits (n = 5, n=25)

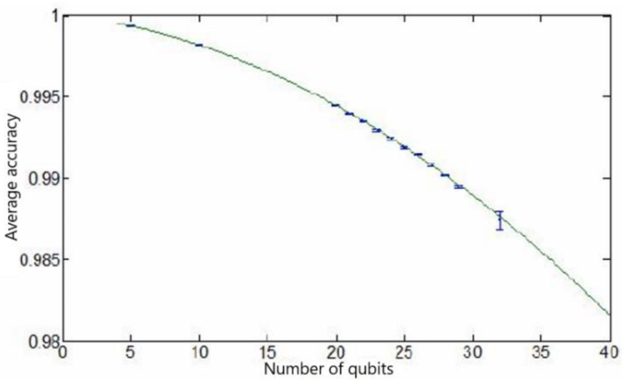

Fig. 3. Accuracy of the algorithm in terms of the number of qubits

Here is a dependence of average accuracy of quantum Fourier algorithm realization on a number of qubits for noise level e = 0.01. For a given number of qubits a certain number of runs of the algorithm have been performed.

5. Development of a scheme for correcting arbitrary errors

Let's consider a more general situation when all possible one-qubit errors are in effect in the system. To fix all kinds of errors (X, Y and Z) in one qubit, you must use a n qubit. Let us define the minimal n. The 2^n-dimensional space of the n qubit system must not be less than 2(3n + 1)-dimensional error space $2^{n-1} \geq 3n+1$. It is seen from inequality that $n_{\min} = 5$. The effect of noise on a system of 9 qubits can be described using equations. The action of the environment is presented in the form of a super operator:

$$\$_{12..9}\, p^{(9)} = (1-\frac{3p}{2})\cdot \$_{12..8}\, p^{(9)} + \frac{p}{2}(IIIIIIIIX \cdot \$_{12..8}\, p^{(9)} \cdot IIIIIIIIX + \\ + IIIIIIIIY \cdot \$_{12...8}\, p^{(9)} \cdot IIIIIIIIY + IIIIIIIIZ \cdot \$_{12...8}\, p^{(9)} \cdot) IIIIIIIIZ. \tag{8}$$

Equation (8) reflects the sum of quantum states of a system of 9 qubits, where p is the noise factor (from 0 to 1), $\$p$ describes bit errors during interaction with the environment. Syndromes (I, X, Y, Z) are used to correct errors. The first syndrome determines whether the 1st and 2nd qubits are equal, the second syndrome answers the question of the equality of the 2nd and 3rd qubits, etc. Knowing the results of the syndromes, we will find out which of the three qubits is wrong. Syndromes are measurements that result in the loss of some off-diagonal elements of the density matrix. The reduction will eliminate all probabilities corresponding to transitions from states with equal qubits to states with unequal qubits, as well as the probabilities of transitions from states with unequal to states with equal qubits. The first syndrome nullifies the elements marked with X and so on. The correction algorithm first corrects the bit error in each group of three qubits. This operation thereby reduces the noise of the bit component, that is $p'_{\text{bit}} < p_{\text{bit}}$, but this leads to an increase in the noise of the phase component p(phase) and the mixed error component $p_{\text{bit+phase}}$. The characteristic increase in the measure of decoherence caused by the correction of the bit error with the admitted phase error is shown in Fig. 4 by dots. The solid line shows the decrease in the measure of decoherence (at small errors) with the admitted bit error and the dotted line marks the case without the correction procedure. The graphs of the dependence of μ on p described below have been modeled on the developed model of a quantum computing device (Fig. 1). Using 100 runs of Shor's algorithm and comparing the results obtained for the presence and value of an error depending on the number of qubits and other input data.

Using this approach, by reducing the noise according to the principles of quantum mechanics, and increasing noise, one can find the approximate behavior of the curve $\mu(p)$. This graph refers to the case when the phase deteriorates first, and then its recovery. The opposite case is shown in Fig. 5. Conclusion: if we know that bit errors are the most complex in the system, then we should first correct them, and only then phase errors. Dependence $\mu(p)$ as a whole segment is shown in Fig. 5. In this scheme, one has to use syndromes that require the use of additional qubits. The result of simulation this correction scheme is the dependence $\mu(p)$ is shown in Fig. 6.

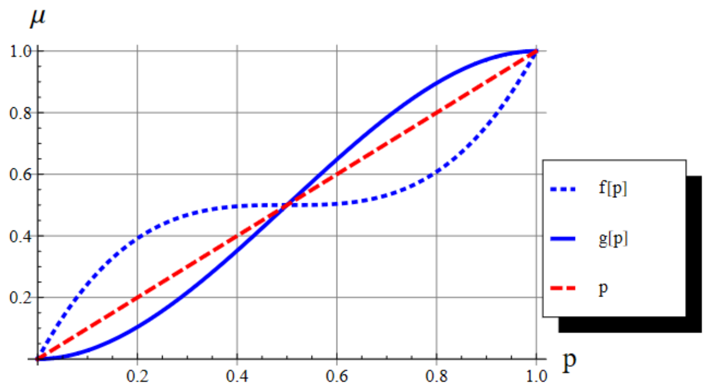

Fig. 4. Typical change when correcting various errors

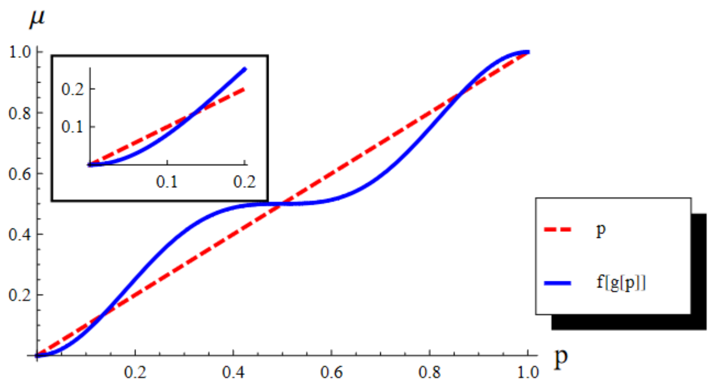

Fig. 5. Dependence $\mu(p)$ on the whole segment

It is similar to the coding schemes already considered. The code correction scheme consists of an encoding scheme, a noise area, a correction and decoding scheme (Fig. 7). $|\psi\rangle = a|0\rangle + b|1\rangle$ is represented by a 5-qubit state $|\psi'\rangle = a|c_0\rangle + b|c_1\rangle$, where $|c_0\rangle = |00000\rangle + |11000\rangle + |01100\rangle + |00110\rangle + |00011\rangle + |10001\rangle - |10100\rangle - |01010\rangle - |00101\rangle - |10010\rangle - |01001\rangle - |11110\rangle - |01111\rangle - |10111\rangle - |11011\rangle - |11101\rangle$, $|c_1\rangle = |11111\rangle + |00111\rangle + |10011\rangle + |11001\rangle + |11100\rangle + |01110\rangle - |01011\rangle - |10101\rangle - |11010\rangle - |01101\rangle - |10110\rangle - |00001\rangle - |10000\rangle - |01000\rangle - |00100\rangle - |00010\rangle$.

We can encode with such codewords using the scheme. The decoding scheme is obtained by transposing the coding scheme. The correction procedure is shown in Fig. 4. For correction, four syndromes have been used; in this case they are implemented as four ancilla qubits. The measurement results of which influence further correction P. Moreover, $k = n - 1$. In the case of bit error correction, two syndromes have been used, in the case of the code, four syndromes have been used.

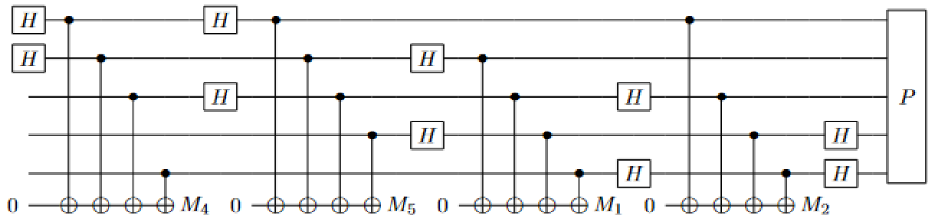

Fig. 6. The scheme of the fix in the code

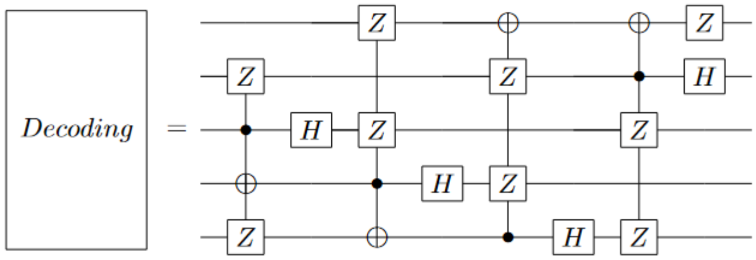

Fig. 7. Decoding scheme in code

The algorithm will correct errors in the effect of noise on an arbitrary pure state at $p < p_c = 0.093$. The dependences of the decoherence [8] measure on the noise level at small p are well approximated by the $\mu \cong 3p^2$ parabola. The dependences $\mu(p)$ for the code can also be replaced by quadratic at low noises $p < p_c$. For a 9-qubit code $\mu \cong 35p^2$, $p \to 0$, and for a 5-qubit code $\mu \cong 15p^2$, $p \to 0$. The measure of decoherence (Fig. 8) can be represented in the form $\mu = bp^2$, where b varies weakly for low noise levels and decreases for all error correction algorithms.

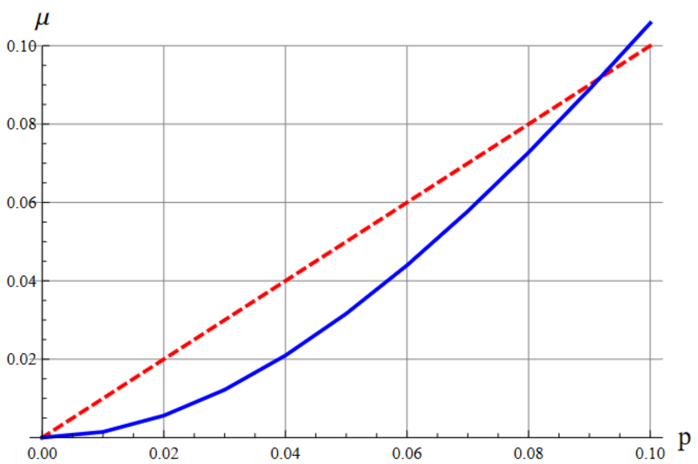

Fig. 8. A measure of decoherence in a code

6. Conclusion

This paper describes the basics of developing quantum algorithms and modeling entangled quantum computations applicable to quantum algorithms. Ansatz of the work is developed codes for correcting various types of errors. Quantum algorithms involve the use of vector and matrix algebra. The structure of the quantum computer model is determined and reflects all its functional features, advantages and disadvantages. Implementation of the scheme for correcting the main types of quantum errors based on the basic scheme for constructing correction codes have been done. The types of noise and their influence on the type of density matrix with a particular dimensionality are described and analyzed. The significance of the study lies in the development of a scheme for correcting arbitrary errors and carrying out experiments on the developed software model of a quantum computing device. Codes for correcting various types of errors are numerically simulated.

Correcting errors is one of the major challenges considering quantum computing devices. Moreover, without solving this problem, further successful developments in this promising area will become ineffective. In this paper, the codes for correcting various types of errors are numerically simulated. The main obstacles and difficulties in the way of protecting the channel from noise are analyzed. Some methods of overcoming them are proposed. Implementation of schemes for correcting two main types of quantum errors has been made. The dependences of the data distortion on the noise level and the decoherence measure on the noise level in one qubit are demonstrated. The effect of different types of quantum noise on some types of quantum gates and on the result of a particular quantum algorithm is described. The growth of the error in the simulation results along the dimensionality exponent as a function of the number of qubits is revealed.

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