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Neural Networks in Engineering Design: Robust Practical Stability Analysis

Trayan Stamov

Technical University of Sofia, Department of Machine Elements and Non-metallic Constructions, "Sveti Kliment Ohridski" 8, 1756 Sofia, Bulgaria
E-mail: tstamov@tu-sofia.bg

Abstract: *In recent years, we are witnessing artificial intelligence being deployed on embedded platforms in our everyday life, including engineering design practice problems starting from early stage design ideas to the final decision. One of the most challenging problems is related to the design and implementation of neural networks in engineering design tasks. The successful design and practical applications of neural network models depend on their qualitative properties. Elaborating efficient stability is known to be of a high importance. Also, different stability notions are applied for differently behaving models. In addition, uncertainties are ubiquitous in neural network systems, and may result in performance degradation, hazards or system damage. Driven by practical needs and theoretical challenges, the rigorous handling of uncertainties in the neural network design stage is an essential research topic. In this research, the concept of robust practical stability is introduced for generalized discrete neural network models under uncertainties applied in engineering design. A robust practical stability analysis is offered using the Lyapunov function method. Since practical stability concept is more appropriate for engineering applications, the obtained results can be of a practical significance to numerous engineering design problems of diverse interest.*

Keywords: *Neural networks, engineering design, practical stability, uncertainties, robustness.*

1. Introduction

The great progress in the development of artificial intelligence methods affects every sphere of life. The necessity of using such methods and technologies arises naturally in a wide variety of tasks such as integrating information, analysing data, and improving the decision-making.

Advancements in artificial intelligence methods and approaches expand the opportunities for neural networks applications, and hence, artificial neural network systems become widely used models in engineering. Neural networks can be successfully applied in modelling, pattern recognition, optimization, classification, forecasting, estimation and much more. See, for example, [1-3] and the references therein. In recent decades, there has been a substantial growth of research interest in the development and applications of neural networks comprising engineering design. For example, in the paper [4] a specific type of neural networks is proposed in interior design to identify products in scenes and finding stylistically similar products. In [5] artificial neural networks are used for the design of a manufacturing system and providing an efficient decision-making framework for selecting the best design alternative. In [6] a neural network system is proposed for retrieval of engineering designs. The paper [7] offers a neural network approach to analyse the relationship between product design forms and their images. Authors found that based on the proposed design model, the designer can also grasp the image feeling of the whole product in the idea development stage. This is due mainly to the key advantages of neural networks such as learning ability, storage ability, fault tolerance, inductive ability, parallel handling ability. In [8] an approximation method for engineering design problems is presented using neural networks. The paper [9] is devoted to the application of neural networks in the design of the exterior form of running shoes. More results on the applications of different classes of neural network systems in engineering design can be found in [10-12]. Some very recent results are presented in [13-15].

Since the efficient applications of a neural network model in engineering design depend on its dynamic behaviour; the qualitative analysis of the dynamic behaviours is an essential step in the practical design of the neural networks.

Stability of the states is known as one of the most important problems in the qualitative analysis of neural networks. In engineering design problems, achieving stable steady states is crucial for many applications since such states are the feasible design solution [16, 17]. For example, if a neural network is used to solve optimization design problems, the steady state represents the optimal solution [8, 10]. In pattern recognition problems, the equilibria are pattern [5, 6]. If a steady state is not stable, then small variations in the initial data (inputs, initial design ideas) may lead to huge perturbations in the output values.

The most studied stability type is the asymptotic stability of the states [18-22]. This is due mainly to the fact that it guarantees the fastest convergence rate. In the case when the neural network is used to solve pattern recognition problems, the global asymptotic stability guarantees a very fast recognition of the patterns (even in case when some initial information is missing). In case of solving optimization problems, the global asymptotic stability guarantees a fast approaching to the optimal solution (the best design concept) independently of the initial data [23].

However, the concept of practical stability is considered to be more appropriate in numerous practical engineering problems [24-26]. In fact, in many cases, though a system is stable or asymptotically stable in the classical mathematical (Lyapunov) sense, it is actually useless in practice because of undesirable characteristics [27-29]. Also, for practically stable models, the system may not be stable mathematically, but can oscillate close to the desired state, in which the performance is still acceptable. In addition, the practical stability properties are useful for models with multi-stable dynamics [30], as well as, in the cases when not only the qualitative behaviour but also the quantitative data, such as specific trajectory bounds are of importance [26].

However, most of the contributions to the practical stability theory for neural network models are related to deterministic models. The practical stability results for discrete systems are relatively few [31-33].

On the other hand, due to the modelling and product information inaccuracies, errors in measurement or random errors, parametric uncertainty occur in the neural network models. Such uncertainties may affect the stability, and therefore, may degraded the performance of the system and the design quality. Hence, robust stability analysis of neural networks is an important part of their qualitative analysis. Since robust design methods may greatly contribute to increase the product quality, recently the robustness in engineering design systems based on neural networks is a very hot research topic [34-37].

One of the most applied method in stability analysis of neural network systems is the Lyapunov function method [24, 38, 39]. It is applied to continuous [19-23, 28, 29] as well as to discrete models [18, 31-33, 40, 41]. The application of the method is based on the use of an auxiliary function (Lyapunov function) with specific properties. No knowledge on the solutions are necessary. Due to the simplifications offered in the applications, the method is not losing its popularity today [42].

For robust stability analysis, researchers have actively developed efficient robust design methods in order to reduce the unstable and/or inconsistent data results arising from the uncertainties and achieve an optimal design. Among all methods, the Lyapunov function method has been successfully applied to investigate the robustness in a variety of models with uncertain terms [43-47], including models in engineering such as robotic manipulators [48, 49], high speed rotors [50], aircraft and aerospace systems [51, 52]. In [53] the Lyapunov-based approach is applied in the lithium-ion batteries accurate state estimation in order to maintain accuracy and robustness. In most of the results, the Lyapunov method is again applied to deterministic models. There are not so many results for robust stability of discrete models [54-57].

Despite the high importance of the robust practical stability notion it has not been developed for discrete neural networks or for specific models used in engineering design, which is the basic aim of the paper.

In this paper, motivated by the above discussion we will introduce the hybrid concept of global robust practical exponential, defined for a class of generalized discrete neural networks applied in engineering design tasks. By using the Lyapunov function method, some sufficient conditions that guarantee the defined stability behaviour are established. The proposed results extend and complement some existing stability criteria for discrete neural networks and can be used in the robust product design process to help designers search for an optimum combination of variable characteristic values for a given product design problem.

2. The model

A network model, in general, is a collection of nodes (vertices) joint in pairs by edges (links). The connection between the nodes is organized into a logical sequence of layers. At the start of the sequence, a neural network applies a layer of functions to a set of input variables. Subsequent layers take as input the output of some of the functions in the previous layer. Finally, the last layer in the network maps to the output data. Indeed, much of today's engineering design work consists of running design variables (inputs) x and receiving output of responses (outputs) y. The input variables represent initial design concepts or input design variables in different design tasks. The output variables describe the final design solution (final design concepts or the optimal design solution).

Several neural network models with an architecture represented in Fig. 1 have been applied in the study of the product form of mobile phones [12] to determine the best combination of product form elements for matching a desirable product image.

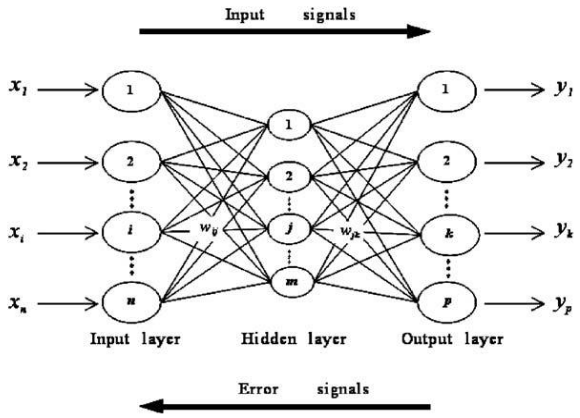

Fig. 1. A three layers neural network [12]

The inputs variables (inputs neurons, x) in most of the proposed neural network models represent top shape, body shape, bottom shape, length and width ratio of body, function buttons style, number buttons arrangement, screen size, screen mask and function buttons and outline division style, and the output variables y are the desirable product images. In some of the proposed neural network models not all input variables have been used. The type of function and the connection between layer outputs and inputs defines the neural network topology. The variables in the output layer in [12] have been obtained by the model

(1) $$y_k = f\left(\sum_j w_{jk} x_j - \theta_k\right), \quad 1 \le k \le p,$$

where the authors used a sigmoid activation function $f(t) = \frac{1}{1+e^{-t}}$, θ_k are threshold values, w_{jk} represent the weights for the connection between neuron $j,\ j = 1, 2, \dots, m$ and neuron $k,\ k = 1, 2, \dots, p$, respectively.

The model (1) has been generalized in [18] to the discrete-time neural network system,

(2) $$x_i(k+1) = -\,a_i x_i(k) + \sum_j w_{ij} g_j(x_j(k)) + J_i,$$

where: $1 \le i,\ j \le n, n$ is the number of the units (nodes, neurons, respectively) in the neural network model (2); $x_j(k)$ is the state of the input j at the discrete time k; $k = 0, 1, 2, \dots$, w_{ij} are the connection weights; $g_j(x_j(k))$ denotes the activation function of the neuron $j,\ j = 1, 2, \dots, n$; J_i represents i-th external bias of the neuron

j. By adding the constants a_i, the model (2) takes into an account the opportunity of the neuron $i,\ i = 1, 2, \ldots, n$, to resets its potential to the resting state when isolated from other nodes and inputs with a constant rate a_i. This is an important requirement for a neural network model to reach steady states.

It is well known that uncertainties can often lead to a trial and error approach to design, where a designer may never uncover the functional relationship between

$$x(k) = \big(x_1(k), x_2(k), \ldots, x_n(k)\big), \text{ and}$$

$$y = x(k+1) = (x_1(k+1), x_2(k+1), \ldots, x_n(k+1)),$$

and therefore may never identify the best settings for input values. Therefore, investigating neural network models with uncertainties that can be applied in engineering design tasks is important for applications and a challenging issue.

In this paper, in order to study the effects of uncertainties on the practical stability behaviour of the model (2), we will consider the following neural network system with uncertainties

(3) $$x_i(k+1) = -(a_i + a_i^*)x_i(k) + \sum_j (w_{ij} + w_{ij}^*) g_j(x_j(k)) + J_i + J_i^*,$$

where: $1 \le i, j \le n$; a_i^* denote unknown bounded parameters in the constant rate a_i; w_{ij}^* are the constant uncertain parameters in the connection weights; J_i^* represents the uncertainty in the external input of the *i*-th unit.

3. Robust practical stability definitions

We will denote by $x(k) = x(k; k_0, x_0)$, $x(k) = (x_1(k), x_2(k), \ldots, x_n(k))$, the state of the model (2) at the time (step) k with initial data

$$x(k_0) = x_0,$$

where: k_0 is some initial time; $k, (k \ge k_0), k_0$ are nonnegative integers (at the first step $k_0 = 0$); $x_0 = (x_{01}, x_{02}, \ldots, x_{0n})$ represents the original sequence of data [12].

In this paper, the norm of the *n*-dimensional vector

$$x(k) = (x_1(k), x_2(k), \ldots, x_n(k)),$$

will be defined as

$$||x(k)|| = \sum_{i=1}^{n} |x_i(k)|, \quad k = 0, 1, 2, \ldots$$

Some global exponential stability criteria have been proposed in [18] for the steady state of the model (2). The goal of this research is to establish robust practical stability results for the model (2) investigating the effects of the uncertainties. To this end, following [24, 28, 31, 32, 33] we will introduce the following definitions.

Definition 1. The model (2) is said to be *globally practically exponentially stable*, if there exist constants $0 < \lambda < 1$, $M \ge 0$ and $A > 0$ such that

$$||x(k)|| \le M\, ||x(0)||\lambda^k + A, \quad k = 0, 1, 2, \ldots$$

Remark 1. A comparison between Definition 1 for global practical stability of system (2) and Definition 1 in [18] for global exponential stability of a steady state of the system (2) again shows that both concepts are quite independent, and in general, neither imply nor exclude each other. The practical stability concept is more useful in engineering since it can be achieved in a setting time which can highly improve the effectiveness in real applications [24-29, 31-33]. In addition, it can be applied for multi-stable systems [30].

Definition 2. The model (2) is said to be *globally robustly practically exponentially stable*}, if the model (3) is globally practically exponentially stable for any values of the uncertain parameters a_i^*, w_{ij}^* and J_i^*.

The model (2) is known as the "nominate" system for the neural network model (3) [44, 47].

4. Basics of the Lyapunov function method

The use of the Lyapunov function method in the stability and stabilization of systems involves the construction of a nonnegative control Lyapunov function V with specific properties. For continuous time systems, it is required that the derivative of the function with respect to the states of the system under consideration to be negative [19-23, 28, 29]. For discrete time systems, such a requirement is considered for the difference between two consecutive values of the function [18, 31-33, 40, 41].

In this paper, we will use Lyapunov functions $V(k,x(k)) > 0$ for any $k = 0,1,2,...$ The difference between two consecutive values of the function will be denoted by

$$\Delta V(k,x(k)) = V(k+1,x(k+1)) - V(k,x(k)),\ k = 0,1,2,...$$

The following result from [33] will be useful in our robust practical stability analysis.

Lemma 1. If there exist a Lyapunov function $V(k,x(k)) > 0$ for any $k = 0,1,2,...$, and positive constants c_1, c_2, c_3, a and ρ, such that $c_3 < c_2$,

(i) $c_1||x(k)|| \leq V(k,x(k)) \leq c_2||x(k)|| + a,\ k = 0,1,2,...$,

(ii) $\Delta V(k,x(k)) \leq -c_3 V(k,x(k)) + \rho,\ k = 0,1,2,...$,

then

$$V(k,x(k;0,x_0)) \leq V(k,x(0))\sigma^k + \frac{\alpha_1}{1-\sigma},$$

where $\sigma = 1 - \frac{c_3}{c_2}$, $\alpha_1 = \frac{a c_3}{c_2} + \rho$.

5. Global robust practical stability analysis

We will study the global robust practical exponential stability of the neural network model (2) under the following assumptions on the system's parameters:

A1. Any system output $x(k)$ can be measured and its initial values are assumed to be in a compact set.

A2. The activation functions g_i are such that functions $g_i(0) = 0$ and

$$|g_i(u) - g_i(v)| \leq L_i\, |u - v|,$$

for any real numbers u and v, where L_i are positive constants for any $1 \leq i \leq n$.

A3. The constants a_i, w_{ij} and J_i are real numbers for $1 \leq i,\ j \leq n$.

A4. The uncertain constants a_i^*, w_{ij}^* and J_i^* are real numbers for $1 \leq i,\ j \leq n$.

Theorem 1. Assume that:

(i) Assumptions A1-A4 hold;

(ii) The uncertain parameters a_i^*, w_{ij}^* and J_i^* are bounded, $a_i + a_i^* > 1$, and there exist positive constants c_3 and ρ, $c_3 < 1$ and $\rho > 0$ such that

(4) $$\min_{1\leq i\leq n}(a_i + a_i^* - 1) - \max_{1\leq j\leq n} L_i\left(\sum_j(|w_{ji}| + |w_{ji}^*|)\right) \geq c_3,$$

(5) $$\sum_{i=1}^{n}(J_i + J_i^*) < \rho \text{ for any } 1 \le i, j \le n.$$

Then the model (2) is globally robustly practically exponentially stable.

Proof: Let $x(k)$ be the system (2) output with initial data $x(0)$, that belong to a compact set.

Consider a Lyapunov function of the type

(6) $$V(k, x(k)) = ||x(k)|| + a \quad = \sum_{i=1}^{n}|x_i(k)| + a.$$

The function (6) satisfies condition (i) of Lemma 1 for some positive constants c_1 and c_2 $(c_2 \ge 1)$.

For the difference between two consecutive values of the function with respect to system (3), we get

$$\Delta V\big(k, x(k)\big) = V\big(k+1, x(k+1)\big) - V\big(k, x(k)\big) = \sum_{i=1}^{n}(|x_i(k+1)| - |x_i(k)|) =$$
$$= -\sum_{i=1}^{n}(a_i + a_i^* - 1)|x_i(k)| + \sum_{i=1}^{n}\sum_j(|w_{ij}| + |w_{ij}^*|) \quad |g_j(x_j(k)| +$$
$$+\sum_{i=1}^{n}((J_i + J_i^*)).$$

From the above estimate, applying A2 and conditions (4) and (5) of Theorem 1, we have

$$\Delta V\big(k, x(k)\big) \le -\min_{1\le i\le n}(a_i + a_i^* - 1)\sum_{i=1}^{n}|x_i(k)| + \sum_{i=1}^{n}\sum_j(|w_{ij}| +$$
$$|w_{ij}^*|)L_j|x_j(k)| + \rho \le$$
$$\le -\min_{1\le i\le n}(a_i + a_i^* - 1)V(k, x(k)) + \max_{1\le i\le n} L_i\big(\sum_j |w_{ji}|\big)V(k, x(k)) \ + \rho,$$

(7) $$\le -c_3 V\big(k, x(k)\big) + \rho, \ k = 0, 1, 2, \dots$$

Since the constant c_3 in (7) is such that $c_3 < 1$, and $c_2 \ge 1$, then $c_3 < c_2$ and condition (ii) of Lemma 1 is satisfied.

From Lemma 1, we have

$$V(k, x(k; 0, x_0)) \le V(k, x(0))\sigma^k + \frac{\alpha_1}{1-\sigma},$$

where $\sigma = 1 - \frac{c_3}{c_2}, .\alpha_1 = \frac{ac_3}{c_2} + \rho.$

Hence,

$$||x(k; 0, x_0)|| \le ||x(0)||\sigma^k + \frac{\alpha_1}{1-\sigma},$$

which proves that the model (2) is globally robustly practically exponentially stable.

The result in Theorem 1 can be generalized, if instead of constants we use functions of the Hahn class K of continuous and strictly increasing functions that are zero at zero.

Lemma 2. If in Lemma 1, the constants c_1, c_2, c_3 are replaced by functions $c_1(r), c_2(r), c_3(r)$ from the class K, then its assertion remains true.

Theorem 2. If in Theorem 1, the constant c_3 is replaced by a function $c_3(r)$, $0 < c_3(r) < 1$, from the class K, then the model (2) is globally robustly practically exponentially stable.

Remark 2. Theorems 1 and 2 offer efficient global robust practical stability criteria for a neural network model (2) used in engineering design. In fact, practical stability is one of the most important aspects of the stability theory and applications of neural networks. Despite the great possibilities for application, the robust practical stability concept has not been applied to neural networks of type (2). With this research, we extend and improve some existing practical stability results for discrete-time neural networks [39-41] to the robustness case. The obtained results can be also used for different types of discrete neural networks applied in engineering design tasks.

Remark 3. The new feasible conditions in Theorems 1 and 2 are formulated in terms of the system's parameters and can be easily checked in particular applications. In addition, in the case of global practical exponential robustness, the desired stability behaviour can be achieved without any constraints on the initial states (initial design parameters).

6. Discussion

It is well known that neural networks are used as models in engineering design to describe the evolution of a design process.

In different engineering design tasks, the input variables and output variables have different meaning. For example, in the *I*-beam design problem discussed in [8], there are three input design variables x_1, x_2, x_3, where x_1 denotes the web height, x_2 denotes the web thickness, x_3 denotes the area of the flange, and the output is the minimum cross-sectional area design of a welded I-beam (optimal solution). Paper [9] considers a design support system for the exterior form of running shoes using neural networks. The structure of the used model is represented in Fig. 2, where there are 30 input design variables *x*, and 4 output design variables *y*. For the particular meaning of these variables, see [9]. In paper [10] the inputs are the initial design concepts, and the outputs are the final conceptual design alternatives. The authors propose three final conceptual alternatives. Indeed, as the authors in [10] state "selecting the best product concept is one of the most critical tasks in a new product development environment."

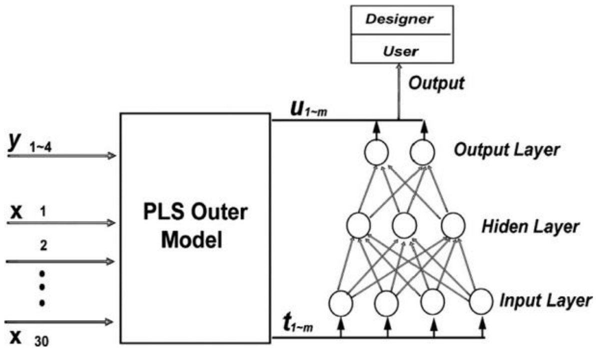

Fig. 2. A neural network model structure used in [9]

For numerous engineering design applications that use the neural network approach, there is not only one steady state [10]. In most of the cases the designers have used multi-stable neural network systems. For such cases, the concept of practical stability is the most appropriate one. For example, in [14] the authors propose a scheme using two neural networks to reduce the computational burden of battery design. The stability of both neural networks is essential.

In addition, making decisions at this stage becomes very difficult due to imprecise and uncertain product requirements. So, the determining the most satisfying conceptual design under uncertainties is a very challenging and important issue. In fact, uncertainties in the neural network parameters can cause unexpected behaviours and variation in performance [34].

In this paper, some global robust practical exponential stability criteria are presented for a discrete-time neural network models used in the product design. The practical meaning of the results being proposed is that if the uncertainties are bounded, and the system's parameter satisfy the conditions of Theorem 1 or Theorem 2, then the corresponding neural network system is globally robustly practically exponentially stable. Since, the parameter variations in neural networks, particularly in their implementation are ubiquitous, our results can be used by designers to avoid decision-making mistakes under uncertainties. We addressed the practical robustness by using the Lyapunov function method and exploring the bounds of the system parameters.

The obtained results and the proposed approach can help designers comprehensively consider design parameters and make fast and accurate design evaluation. Since the proposed robust practical stability concept and the Lyapunov function control technique have a great potential in applications, it is expected that our research will inspire the researchers to apply the proposed approach to different neural networks models of diverse interest.

7. Conclusion

Engineering design is a multiplex process requiring tasks such as decision-making, optimization steps, forecasting, etc. during the elaboration of the product being designed. An efficient solution of these tasks is offered by the neural network systems. The fast development in the artificial intelligence methods, large-scale computers and parallel computations expand the opportunities for neural networks applications in the product design. Knowledge and understanding of these technologies have led to the development of new models, novel methods and extending the existing techniques for analysis of the neural network dynamics.

The stability of the feasible solutions is one of the main tasks in the analysis of the systems dynamics since the use of a stable model leads to a better modelling, ensures enhanced effectiveness of the final design solution and guarantees reaching performance [17].

In this paper, in order to take the advantages of the practical stability concept, a generalized notion of global robust practical exponential stability for neural networks used in engineering design [12, 18] is introduced. By the Lyapunov function analysis efficient new criteria are established that involve inequalities for the systems parameters. Since practical stability is an essential qualitative property for numerous neural network models with multi-stable dynamics, the analysis of such systems is of a considerable interest to more applications. Practical stability is very important in numerous cases when the model can be unstable mathematically, but its performance may be sufficient for the practical point of view. Also, since improving robustness is very crucial in the robust product design process under bounded uncertainties [49] in order to increase the efficiency and effectiveness of product design, our approach can be applied to many engineering problems.

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