BULGARIAN ACADEMY OF SCIENCES

CYBERNETICS AND INFORMATION TECHNOLOGIES • Volume **21**, No 3
Sofia • 2021 Print ISSN: 1311-9702; Online ISSN: 1314-4081
DOI: 10.2478/cait-2021-0033

Bi-Level Optimization Model for Urban Traffic Control

Krasimira Stoilova, Todor Stoilov, Stanislav Dimitrov

Institute of Information and Communication Technologies – Bulgarian Academy of Sciences, 1113 Sofia, Bulgaria
E-mails: krasimira.stoilova@iict.bas.bg todor.stoilov@icct.bas.bg stanislav.dimitrov@iict.bas.bg

***Abstract**: The urban traffic control optimization is a complex problem because of the interconnections among the junctions and the dynamical behavior of the traffic flows. Optimization with one control variable in the literature is presented. In this research optimization model consisting of two control variables is developed. Hierarchical bi-level methodology is proposed for realization of integrated optimal control. The urban traffic management is implemented by simultaneously control of traffic light cycles and green light durations of the traffic lights of urban network of crossroads.*

***Keywords**: Hierarchical modeling, optimization, bi-level modeling and optimization, urban traffic control.*

1. Introduction

Improvement of the traffic management in urban areas for optimizing the transportation behavior is a problem, being continuously solved. There is no need to explain the importance of the problem, related to the traffic control and the consequences for improving the traffic operation, transport services, pollutions and fuel consumptions. From the point of view of the control theory the control influences, which can be applied for the transport control are not so many and they define a control space, which contains the duration of the green lights of a stage (phases or splits) during which the appropriate stream has right of movement; the cycle time which contains all phases and time loss between phases on a junction; and the offset which is the time difference between cycles of successive crossroads and it gives rise to a “green wave” along an artery [35].

The traffic light settings play a general role in determining the quality of operating conditions at signalized intersections. The goal of the traffic lights control is to minimize the average vehicle stay or some other measures of effectiveness [25]. Various models for optimizing the traffic lights control have been developed in research studies. Traffic lights control is formalized for both saturated and oversaturated junctions [8, 64].

Formal developments for the application of different components of the traffic lights control space (split, cycle, offset) have been done as intelligent algorithms for enhancement the operations on signal intersections [3, 34, 36, 51, 61]. Simulation and heuristic assumptions are widely applied for achieving robust and realistic traffic management [22, 52].

An important feature of these researches is that they apply control impact mainly by the split of the traffic lights. Simultaneous optimization of split and cycle is not met. In [13, 14] the traffic control is performed by the split of the lights, assuming constant cycle. In [6, 19] only the cycle is considered without the usage of the split.

The goal of the research is to develop an optimization model, which improves the urban traffic management in order to decrease the queues in front of the junctions. This paper targets the development of a formal model, which allows simultaneous optimization of the split and the cycle of the traffic lights. Thus, the control space of this problem is increased, which allows potentially achieving better control results in the traffic management. The goal of the study is implemented by the application of a methodology from the hierarchical bi-level optimization. The added value of this research is the increase of the control space of the traffic lights control. Such increase is obtained by simultaneously applying as control variables both traffic lights parameters: the green lights durations (the split) and the traffic light cycles. For achieving this research goal several problems have to be solved: 1) overview of the traffic optimization models; 2) methodologies and approaches for traffic control modeling; 3) analytical definition of the lower-level optimization problem of the model; 4) analytical definition of the upper-level optimization problem of the model; 5) numerical simulation of the newly developed optimization model. The solutions of these defined problems are sequentially described below.

2. Overview of models optimizing traffic lights green duration (split) and cycle time

One of the important and challenging real-world problems, which aim to minimize the travel time of vehicles by coordinating their movement at the road intersections is traffic signal control. The macroscopic models consider a behavior of aggregate traffic flow. This means that no distinction is made of user-classes, such as traveler types (commuters, freight, tourist, etc.), vehicles types (person-cars, trucks, busses, and vans), paying and non-paying traffic, and various types of guided vehicles [23]. This aggregation increases the applicability of macroscopic models to the synthesis and analysis of multilane multiclass traffic flow. For a review of macroscopic modelling approaches used for traffic networks one can refer to [33]. The modelling of both main network's components – links and nodes are analyzed with their advantages and disadvantages and solution procedures are discussed. The macroscopic traffic control of a large-scale mixed transportation network consisting of freeway and urban network is considered in [42]. The urban network is presented for two regions, where each one has a well-defined macroscopic fundamental diagram or determined relationship between network density and outflow.

Integration of microscopic and macroscopic modelling for autonomous vehicles is developed in [54]. A microscopic dynamics is considered in junctions at the first level. The second level (macroscopic model) aims to optimize the network performance by minimizing the queues in all road links. The traffic optimization is performed in a global optimization problem. A recent survey of widely acknowledged transportation approaches for traffic signal control is done in [56]. The key point of the survey is the inclusion and application of methods, belonging to artificial intelligence techniques like a variation of machine learning technique called reinforcement learning. Existing traffic signal control systems follow pre-defined signal plans so there is not enough training data to differentiate good and bad traffic signal plan strategies, which are the base of typical supervised learning. The essence of their approach is to change the signal plans and then learn from the outcomes of the reinforcement learning by trial-and-error approach.

The trend for application of adaptive traffic control systems and implementations in practice are discussed in [1]. The current control policy concerns continuously monitoring of the traffic conditions on urban roads and adjusting traffic signal timings to minimize stops and delays. The assessment of the adaptive control strategies is mainly performed by software simulation tools. An overview of some of the widely used micro-simulation packages is given in [40].

The traffic signalization received a great evolution from the first pre-fixed signals with fixed times to the real-time traffic signalization. A roughly taxonomy of traffic signal systems can be found in [55]. Traffic signal control is an important real-world problem, which aims to minimize the travel time of vehicles by controlling their movements in urban environments. The traffic signal control systems use approximate oversimplified information about the traffic dynamics. The more data and complex control methods always require more computing power for implementation of intelligent transportation. For an up to date survey of the problems in traffic control one can refer to [56].

The queue problem of the traffic control is the integration of the different types of control, split, cycle and offset in a common optimization problem. Because such problem will suffer for considerably computational resources, the approach, which is used in the traffic control, is to decentralize the control and to apply only one of the possible control impacts: split or cycle. The tendency to derive models, which target decomposition of the traffic control problem from centralized manner of solution to decentralized one, is explicitly seen. This reduces the workload of evaluations and allows the system to cope with the dynamic changes of the traffic conditions. Centralized and decentralized traffic lights control is developed and analyzed in [9]. The duration of the green lights (split) is the control set in an optimization problem. The strong point of the investigation is the decentralization of the split evaluation for decreasing the computational workload of the traffic control. The same approach for decentralized traffic lights control is applied in [32]. In [62] a decentralized control model for connected automated vehicle trajectory optimization at an isolated signalized intersection with a single-lane road is formalized. Each connected automated vehicle targets the minimization of its own travel time, fuel consumption and safety risk.

The split as the only control impact is used in most researches. In [63] a piecewise linear model of traffic flow dynamics at signalized intersection is derived. Together with non-signalized intersection, a traffic lights control problem is formulated like a mixed integer programming problem. Green splits of the phase control are the control impacts, assuming constant values of the cycles. In [24] the signal timing like split parameters, optimized in central way in a common problem are decomposed by a decentralized approach. Thus, the local signal timing optimization problem controls the timing of only a single intersection. This decomposition reduces the complexity of the centralized control problem and real-time response for the traffic dynamics is achieved. In [9] authors develop and analyze centralized and decentralized traffic lights control. The duration of the green lights (split) is the control set in an optimization problem. The strong point of the investigation is the decentralization of the split evaluation for decreasing the computational workload of the traffic control. The same approach for decentralized traffic lights control is applied in [32].

Attempt to simultaneously control of splits, cycles and offsets are made in [11]. The defined optimization problem in explicit way concerns cycles and offsets of two neighboring junctions, which is very simplified example of integrated traffic control.

The cycle control is also applied for traffic lights control. In [7] only the traffic signal cycle is used as a control variable for minimizing the vehicle delay and increase the throughput on an intersection. In [57] optimization traffic signal cycle for signalized intersections is formalized. The optimization problem evaluates only the durations of the traffic cycles at junctions. The problem takes in consideration the vehicles delay time, pedestrian crossing time, and drivers' anxiety, waiting in front of the traffic lights. Only three intersections are considered.

In [41] simultaneous optimization of the split and cycle is performed for a single junction. The approach is promising but the scale of urban network to only one junction has limited practical applications. The same claims for simultaneously control of split, cycle and offset are described in [27]. But the presented formal model calculates only the green split for four intersections. The defined problem is integer binary one and model predictive control is applied. This means that an integer optimization problem is solved sequentially, but only the first discrete of the solutions are applied in traffic control.

The multicrieria optimization insists on simultaneous optimization according to a set of criteria [18, 37-39]. As a particular application of the multicriteria optimization, this research uses the bi-level hierarchical optimization. The latter applies simultaneously two criteria in hierarchical sequence for the problem optimization.

Each optimization level evaluates its own set of control impacts. But the overall bi-level problem evaluates an extended control set, which contains the overall sum of controls for both hierarchical levels [44]. This potential of the hierarchical optimization can benefit the simultaneous usage of the traffic impacts (split, cycle and offset) by defining and interconnecting optimization problems in hierarchical order. Unfortunately, the hierarchical optimization needs global solutions on each level, which is computationally time consuming. That is the reason currently only the bi-level optimization forms to be applied for more realistic control problems. In [21] a bi-level model for traffic network signal control is formulated as a dynamic Stackelberg game. The optimization problem has equilibrium constraints, which are form of additional optimization problem. The lower level problem applies user equilibrium relations of LVR model [31, 43]. The upper-level control variables are the green splits and the objective is to minimize the travel cost. In [5] the bi-level formalization is applied for the decrease the transportation risk of hazardous loads. The low level problem evaluates the optimal route of transport and the upper level satisfies the community constraints for minimizing the risk. The transport of Liquefied Petroleum Gas (LPG) is of serious risk as the LPG is a hazardous material (hazmat). It is transported in cylinders by vehicles on urban road networks to meet varying demand of community safety. A bi-level optimization of transportation is presented in [26]. The bi-level optimization is applied to stimulate intermodal transport by determining pricing strategies [53]. In [52] the bi-level optimization is applied for bus lane optimization. The simultaneous existence of freeway and bus transportation is considered. Many urban centers have traffic control strategies, based on time-of-day intervals. In [16] a bi-level optimization model for simultaneously solution of the segmentation problems and the traffic control problems over these time intervals is formalized. In [29] coordinated control of the traffic signal which considers the pedestrian crossing delay is studied. Only the split is evaluated on the areas close to metro stations, where many people use the metro station simultaneously with the vehicle traffic.

The form of Stackelberg game is also used for the optimization of vehicle routing problem [58]. The upper-level optimization targets minimization of the carbon emissions from the government's perspective and the lower-level model from the firm's perspective. The route choice in bi-level optimization is given also in [50].

The bi-level optimization is applied in [15] and description of the components of the bi-level problem is given. Definitions, classifications, objectives, constraints, network topology decision variables, and solution methods, related to the urban transportation network design problem is given. This taxonomy presentation gives a picture of transportation network design problem, allowing comparisons of formulation approaches and solution methods of different problems in various classes of urban transportation network problems, defined in the terms of bi-level optimization. In [10] the traffic control problem is formalized also as a bi-level optimization problem. The key point is the choice of methods for solving such problems. Four approaches for solving the bi-level problem are presented: by penalty function, solutions of sequences of quadratic and/or linear sub-problems. In [20] the bi-level programming approach is applied for evaluation of the splits of traffic lights. Both upper and lower level optimization problems evaluates the splits, but under different constraints. The upper level problem applies objective function to maximizing the weighted trip of vehicles. The lower level problem optimizes the traffic assignment. The bi-level formulation of the traffic control problem in [30] evaluates the splits on the upper level by minimizing the average travel time of drivers. The lower level problem goal evaluates and keeps the network's equilibrium.

3. Methodologies and approaches for traffic control modelling

The main methodologies used for the traffic control modelling are two. The first is traffic dynamic following the continuity of flows. It is based on the analogy between the liquid flow dynamics and traffic flow dynamics. This methodology is introduced by Lighthill and Whithan [31], and later by Richards [43] and their model is known currently like "LWR" – the first letters of their families. This model is suitable mainly for highway traffic modelling, although it can be used in special cases for urban traffic control [48].

The second methodology for the traffic control modelling which is used in this research is so called "Store-and-forward-modelling". It is one of the first traffic modelling methodologies suggested in [17]. This model finds extensive and wide application by many researchers due to its simplicity [2, 4, 12, 28, 59]. The model formalizes the queue lengths in front of the traffic lights. The following arguments are included in the model. The queue lengths in front of the traffic lights consists of vehicles denoted by $\mathbf{x}(k+1)$ at the moment $k+1$. This value represents the number of vehicles in the previous moment $\mathbf{x}(k)$ to which is added the incoming traffic flow $\mathbf{q}_{\mathrm{in}}(k)$ decreased by the outgoing flow $\mathbf{q}_{\mathrm{out}}(k)$ according to relation

(1) $$\mathbf{x}(k+1) = \mathbf{x}(k) + \mathbf{q}_{\mathrm{in}}(k) - \mathbf{q}_{\mathrm{out}}(k).$$

The incoming flow $\mathbf{q}_{\mathrm{in}}()$ represents the traffic increase by the number of vehicles which come to the junction and they are added to the queue length. We assume this value is known in advance for any junction and it is regarded as initial known parameter for the model. That is why it will be denoted by $\mathbf{x}_{\mathrm{in}}(k)$ in relation (3).

The essential variable of the store-and-forward model is the outgoing flow $\mathbf{q}_{\mathrm{out}}()$ because it decreases the queue length. The outgoing flow depends on the green light duration of the traffic light and the outgoing flow is proportional to the green light duration. However, the value of the green light duration is constrained by traffic flows, which do not have right of movement and the value of the cycle of the lights. As for the vehicles, which do not have right to move their waiting time increase and their corresponding queue lengths will increase. The outgoing flow depends also on the capacity of the junction links, which is quantified by the value $\mathbf{s}$ of the saturation flow in a direction. The saturation flow $\mathbf{s}$ defines how many vehicles can pass through the intersection for a time interval. Hence, the outgoing flow is proportional to the green light duration $\mathbf{u}(k)$ of this phase and the saturation of the flow $\mathbf{s}$,

(2) $$\mathbf{q}_{\mathrm{out}}(k)=\mathbf{s}\mathbf{u}(k).$$

For the definition of the discrete unit of the time, k, the practice is to use k equal to the cycle duration of the traffic lights. In that manner relation (1) is simplified and it holds for every traffic light cycle,

(3) $$\mathbf{x}=\mathbf{x}_0+\mathbf{x}_{\mathrm{in}}-\mathbf{s}\mathbf{u}.$$

Relation (3) simplifies the traffic dynamics at junctions and it is applied in the optimization problem for the formalization of each queue length in a traffic network. Relation (3) which formalizes the "store-and-forward" model is quite simple, which is a prerequisite for real time implementation of the control policies. However, relation (3) contains as control variable only the green light duration (split) of the traffic lights. The cycle as important control parameter and respectively the offsets are not taken into account as control variables in this simplified model of the traffic dynamics. The simultaneous inclusion of cycle and/or offset in the model of the traffic dynamics needs usage of additional modelling techniques.

This research addresses the optimization problem in bi-level hierarchical form. It is expected by hierarchical organization to integrate the control space of the optimization problem with splits and cycles of the traffic lights. The usage of (3) allows a classical, non-hierarchical, optimization problem to be defined for each urban junction. Thus, minimization of the queue lengths in front of each junction can be achieved. But this approach allows obtaining optimal results only for an isolated junction, but not for series of interconnected ones. Because the practical case of traffic control in general addresses urban network with a set of junctions, the optimization problem must improve the traffic in a network of sections. The optimization of an isolated junction can be performed with the usage of relation (3), but for the case of an urban network it is needed to consider also additional relations of the traffic dynamics among neighboring junctions. The outgoing flows of a junction are incoming ones to a set of neighboring junctions.

This research targets the increase of the control parameters with both the splits and cycles. For this case, a bi-level modeling is applied. Respectively, such hierarchical modeling allows to be considered as a control system set of urban intersections, connected in a transport network. Such approach for bi-level optimization of traffic behavior has been applied in our previous publications [45-49]. The main difference now is the goal of the research – to extend the control space with both the splits and cycles of the traffic lights.

An important reason for the usage of hierarchical optimization is the opportunity not to define explicitly in analytical way a global control problem. The hierarchical approach allows such problem to be defined as interconnected low scale problems, which influence each other. The definition of low scale problem is always easier in comparison with the definition of global high scale problem. Because of the complexity of the traffic control problem, the hierarchical approach, applied here, uses a low-level optimization problem, which uses the splits as control variables. The upper-level optimization problem applies the cycles as control variables. Both optimization problems apply different objective functions for optimization the traffic behavior. But the overall control space of the traffic control is extended with simultaneously evaluation of splits and cycles. This extended control space allows two global optimization functions to be satisfied and to consider more constraints, defined by the upper and lower-level optimization problems.

The bi-level optimization has the following type of operation. The low-level problem finds the optimal values (minimum) of its arguments – the queue lengths in front of the junctions and the control influences are the splits (the optimal values of the green light durations), satisfying a set of problem's constraints. The evaluated optimal solutions from the lower level are sent to the upper level problem, where they are considered as fixed parameters. The meaning of the upper-level problem here is defined to evaluate the optimal duration of the traffic light cycles. Then, the optimal values of the upper level problem (cycles) are sent back to the lower level problem where they are used as fixed parameters, for the lower level optimization problem. Thus, by iterative calculations the bi-level optimization problem evaluates optimal control variables both for splits ant cycles of the traffic lights. In that manner, the bi-level optimization problem incorporates in its definition the well-known "store-and-forward model".

4. Analytical definition of the model's bi-level traffic control problem

The paper presents the case of traffic control in an urban region of Sofia, which is a cross point of business regions, residence ones, set of service points for the habitants and represents intensive transportation network. Our previous researches did different models on this transportation region [46] and different definitions of the goal functions of the optimization problem [47, 48]. The current task of this investigation is the extension of the urban network and the integration of splits and cycles in a common control space. The extended network contains eight interconnected junctions, graphically presented in Fig. 1.

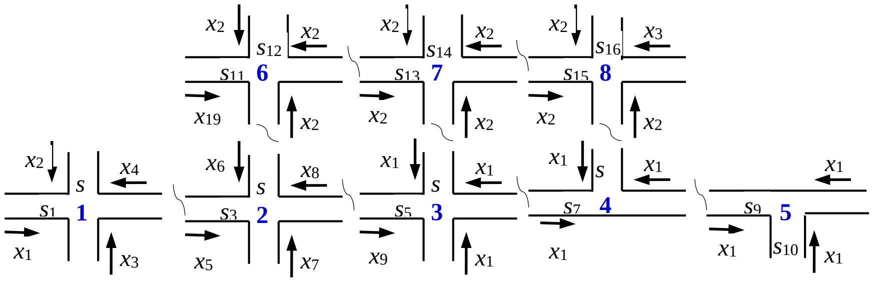

Fig. 1. Traffic network of crossroad sections

The network concerns important part of the traffic scheme in Sofia, including "Shipchenski prohod" boulevard from Universiada Hall till "Assen Yordanov" boulevard and the parallel junctions of "Geo Milev" street from "Nikolay Kopernik" street till "Ivan Dimitrov-kuklata" street.

The initial data for the optimization problem has been estimated by real measurements and appropriate evaluations of the corresponding means for the different days of the week. For the definition of the bi-level optimization problem model complications has been considered concerning more phases of the directions of transportation: straight ahead direction, curve to the right and curve to the left phases at each junction of the network. The single arrow "→", drown for simplicity in Fig. 1 for the traffic flow movement, has to be considered like triple arrows " ".

From statistical estimations the saturation flows for turning ($l_{1,2}$) to the left direction is 0.1 veh per 1 s and turning to the right is estimated to 0.2 veh per 1 s. These values easily can be changed in the derived bi-level model and they do not influence the manner of solution of the optimization problem.

The goal function of the low level problem is chosen to minimize the queue lengths x in all directions by usage the green split as control variable. The scale of the low level problem contains 30 queue lengths x_i, i=1,…, 30, on eight junctions by changing 16 control parameters, u_j, j=1,…,16, which are the splits of the green lights. The goal function of the low-level problem is to minimize the total amount of the queue lengths of vehicles. The "Store-and-Forward model" insists on defining of the initial number of cars x_0 at each junction's direction. These values are assumed to be noisy disturbances for the control. The traffic control concerns the evaluation of such sequence of splits, which will decrease the values of the disturbances x_0. The notations of the saturation flows s_i follow the numeration of the junctions. For example for the first junction the corresponding saturations are noted s_1 and s_2. For the second junction they are respectively s_3 and s_4, etc. The values of saturations of each junction are assumed to be known, s_i, i=1,…,16. The traffic demands are denoted as input traffic flows x_{in}. The initial junctions of the network have been estimated by statistical data. They are assumed as initially given and known for the control problem.

The upper level problem uses the cycles as control impacts in the optimization problem. The traffic lights cycles are denoted by y_j, j=1,…, 8, and they consist of the time for the sequence of green splits, amber (lost time) and red lights for the different phases. It is assumed that the total lost time is fixed to 10% of the value of the cycle of each junction. The notations for the green splits for the first junction are denoted as u_1 and u_2. Respectively, for the total network there are sixteen splits of the green lights durations u_k, k=1,…, 16.

4.1. Analytical definition of the model's low-level optimization problem

The formal description of the low level optimization problem has meaning of minimization of the vehicle queues x_i , i=1,…, 30 by changing 16 splits of the green lights u_k, k=1,…, 16. The goal function is defined in a quadratic form by means to minimize the computational workload. The constraints of the problem describe the store-and-forward relation for each direction of the network junctions. The analytical description of the low-level problem is the following:

(4)
$$\min_{\substack{i=1,\dots,30\\ j=1,\dots,16}} (x_i^2 + u_j^2),$$

subject to

(5)
$$\begin{gathered}
x_1 + (1 + l_1 + l_2)s_1u_1 \le x_{1_0} + x_{1_{\text{in}}},\\
x_2 + (1 + l_1 + l_2)s_2u_2 \le x_{2_0} + x_{2_{\text{in}}},\\
x_3 + (1 + l_1 + l_2)s_2u_2 \le x_{3_0} + x_{3_{\text{in}}},\\
x_4 + (1 + l_1 + l_2)s_1u_1 - s_3u_3 - (l_1 + l_2)s_{14}u_4 \le x_{4_0},\\
x_5 - s_1u_1 - (l_1 + l_2)s_2u_2 + (1 + l_1 + l_2)s_3u_3 \le x_{5_0},\\
x_6 + (1 + l_1 + l_2)s_4u_4 - (l_1 + l_2)s_{11}u_{11} - s_{12}u_{12} \le x_{6_0},\\
x_7 + (1 + l_1 + l_2)s_4u_4 \le x_{7_0} + x_{7_{\text{in}}},\\
x_8 + (1 + l_1 + l_2)s_3u_3 - s_5u_5 - (l_1 + l_2)s_6u_6 \le x_{8_0},\\
x_9 - s_3u_3 - (l_1 + l_2)s_4u_4 + (1 + l_1 + l_2)s_5u_5 \le x_{9_0},\\
x_{10} + (1 + l_1 + l_2)s_6u_6 - (l_1 + l_2)s_{13}u_{13} - s_{14}u_{14} \le x_{10_0},\\
x_{11} + (1 + l_1 + l_2)s_6u_6 \le x_{11_0} + x_{11_{\text{in}}},\\
x_{12} + (1 + l_1 + l_2)s_5u_5 - s_7u_7 - l_1s_8u_8 \le x_{12_0},\\
x_{13} - s_5u_5 - (l_1 + l_2)s_6u_6 + (1 + l_2)s_7u_7 \le x_{13_0},\\
x_{14} + (l_1 + l_2)s_8u_8 - (l_1 + l_2)s_{15}u_{15} - s_{16}u_{16} \le x_{14_0},\\
x_{15} + (1 + l_1)s_7u_7 - s_9u_9 - l_2s_{10}u_{10} \le x_{15_0},\\
x_{16} - s_7u_7 - l_2s_8u_8 + (1 + l_1)s_9u_9 \le x_{16_0},\\
x_{17} + (l_1 + l_2)s_{10}u_{10} \le x_{17_0} + x_{17_{\text{in}}},
\end{gathered}$$

$$x_{18} + (1 + l_2)s_9u_9 \leq x_{18_0} + x_{18_{\text{in}}},$$
$$x_{19} + (1+l_1 + l_2)s_{11}u_{11} \leq x_{19_0} + x_{19_{\text{in}}},$$
$$x_{20} + (1+l_1 + l_2)s_{12}u_{12} \leq x_{20_0} + x_{20_{\text{in}}}$$
$$x_{21} - (l_1 + l_2)s_3u_3 - s_4u_4 + (1 + l_1 + l_2)s_{12}u_{12} \leq x_{21_0},$$
$$x_{22} + (1 + l_1 + l_2)s_{11}u_{11} - s_{13}u_{13} - (l_1 + l_2)s_{14}u_{14} \leq x_{22_0},$$
$$x_{23} - s_{11}u_{11} - (l_1 + l_2)s_{12}u_{12} + (1 + l_1 + l_2)s_{13}u_{13} \leq x_{23_0},$$
$$x_{24} + (1+l_1 + l_2)s_{14}u_{14} \leq x_{24_0} + x_{24_{\text{in}}},$$
$$x_{25} - (l_1 + l_2)s_5u_5 - s_6u_6 + (1 + l_1 + l_2)s_{14}u_{14} \leq x_{25_0},$$
$$x_{26} + (1 + l_1 + l_2)s_{13}u_{13} - s_{15}u_{15} - (l_1 + l_2)s_{16}u_{16} \leq x_{26_0},$$
$$x_{27} - s_{13}u_{13} - (l_1 + l_2)s_{14}u_{14} + (1 + l_1 + l_2)s_{15}u_{15} \leq x_{27_0},$$
$$x_{28} + (1+l_1 + l_2)s_{16}u_{16} \leq x_{28_0} + x_{28_{\text{in}}},$$
$$x_{29} - (l_1 + l_2)s_7u_7 - s_8u_8 + (1 + l_1 + l_2)s_{16}u_{16} \leq x_{29_0},$$
$$x_{30} + (1+l_1 + l_2)s_{15}u_{15} \leq x_{30_0} + x_{30_{\text{in}}},$$
$$u_1 + u_2 = 0.9y_1, \qquad u_3 + u_4 = 0.9y_2,$$
$$u_5 + u_6 = 0.9y_3, \qquad u_7 + u_8 = 0.9y_4,$$
$$u_9 + u_{10} = 0.9y_5, \qquad u_{11} + u_{12} = 0.9y_6,$$
$$u_{13} + u_{14} = 0.9y_{7,} \qquad u_{15} + u_{16} = 0.9y_8.$$

Additional constraints are added in the problem, which consider that the green splits have to be restricted by the traffic lights cycles, denoted by the variables y_j, j=1, …, 8. Latter are evaluated as solutions of the upper level problem, but they are fixed parameters for the lower level problem.

4.2. Analytical definition of the model's upper-level optimization problem

The upper level optimization problem is defined to minimize the durations of the traffic light cycles. This goal is applied, because increasing the traffic cycles leads to increase of the waiting time for vehicles, which do not have right to move. Thus, the increase of the traffic cycle benefit only parts of the vehicle flows. This results in non-fair control policy for the overall transportation flows. The traffic light cycle is also constrained to lower and upper bounds $y_{\min} \leq y \leq y_{\max}$, which frequently are established by legislative requirements. For the current version of the upper level optimization problem the lower and upper bounds have been established to $y_{\min}$ = 40 s and $y_{\max}$ = 100 s. Additional set of constraints is defined according to the solutions of the low-level problem. The queue lengths x_i, i=1,…, 30, and the durations of green splits u_k, k=1,…,16, are used as known parameters for the upper level optimization problem. The analytical form of the upper level optimization problem is defined as

(6)
$$\min_y y^{\mathrm{T}} y,$$
$$y_{\min} \leq y \leq y_{\max},$$
$$u_1 + u_2 = 0.9y_1, \qquad u_3 + u_4 = 0.9y_2,$$
$$u_5 + u_6 = 0.9y_3, \qquad u_7 + u_8 = 0.9y_4,$$
$$u_9 + u_{10} = 0.9y_5, \qquad u_{11} + u_{12} = 0.9y_6,$$
$$u_{13} + u_{14} = 0.9y_7, \qquad u_{15} + u_{16} = 0.9y_8.$$

The upper level problem is also defined as linear-quadratic optimization by means to decrease the computational workload in the control process.

5. Simulation and numerical results

The defined bi-level optimization problem has extended control space containing the set of the green splits u_i, i=1,…, 16, and the set of the cycles y_j, j=1,…, 8. The system states are defined by the set of vehicle queues x_i, i=1,…, 30. The goal of the control is twofold: minimization of the duration of the traffic cycles and minimization of the total number of vehicles inside the urban network. The minimization of the duration of cycles allows obtaining fair distribution of the waiting time of the vehicles in the network. The minimization of the queue lengths result in decreasing the total travel time in the network, decreasing of the environmental pollution, improvement of the social activities, etc. Thus, the bi-level control optimization allows more than one goal function to be satisfied and to increase the control space of the solutions, which benefits more requirements towards the traffic behavior.

The bi-level optimization problem (4)-(6) is solved in MATLAB environment. The MATLAB's extension application tool YALMIP is used [65] and the "solvebilevel" function is mainly used for the solving the bi-level problem. The control of the traffic behavior has been performed by sequentially multiple solving (4)-(6). Each solution of the bi-level problem gives the queue lengths and green splits for the current traffic lights cycle. The resulting values of the traffic behavior (queue lengths) are used as initial data for the next traffic light cycle. Thus, the bi-level problem is solved sequentially for each traffic lights cycle. In that manner, the control process is illustrated in several sequences of solutions of the bi-level problem, which corresponds to the application of the bi-level solutions for a sequence of traffic lights cycles. The results of four sequential calculations of bi-level problem are summarized numerically in Table 1.

Table 1. Bi-level solutions

No		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
I iteration	*y*	40	40	40	40	40	40	40	40								
	x	16.9	12.5	8.2	7.5	9.5	4.9	8.7	7	12.7	15.6	10.6	19.5	14.2	17.2	11.1	11.7
		11.8	7.1	11.6	11.4	14.6	15.7	10.6	11.3	5.7	9.1	6.3	13.9	6.3	7.8		
	u	16.5	19.4	17.4	18.5	18	17.9	18.1	17.9	21.2	14.8	17.7	18.2	16.9	19.1	18.9	17
II iteration	*y*	40	40	40	40	40	40	40	40								
	x	14.6	12.4	4.7	6.8	7.4	5.9	4.3	4.9	9.9	21	7.4	29.8	22.9	29.3	16.5	14.9
		11.1	3.1	3.7	9.9	18.1	21.6	0.4	5.1	4.9	13.3	0.6	2.7	6.2	0		
	u	18.2	17.7	19.3	16.6	19.3	16.6	16.2	19.8	18.3	17.7	19.7	16.2	16.0	19.9	17.3	18.6
III iteration	*y*	40	40	40	40	40	40	40	40								
	x	14.6	12.4	4.7	6.8	7.4	5.9	4.3	4.9	9.9	21	7.4	29.8	22.9	29.3	16.5	14.9
		11.1	3.1	3.7	9.9	18.1	21.6	0.4	5.1	4.9	13.3	0.6	2.7	6.2	0		
	u	18.2	17.7	19.3	16.6	19.3	16.6	16.2	19.8	18.3	17.7	19.7	16.2	16.0	19.9	17.3	18.6
IV iteration	*y*	40	40	40	40	40	40	40	40								
	x	14.6	12.4	4.7	6.8	7.4	5.9	4.3	4.9	9.9	21	7.4	29.8	22.9	29.3	16.5	14.9
		11.1	3.1	3.7	9.9	18.1	21.6	0.4	5.1	4.9	13.3	0.6	2.7	6.2	0		
	u	18.2	17.7	19.3	16.6	19.3	16.6	16.2	19.8	18.3	17.7	19.7	16.2	16.0	19.9	17.3	18.6

The results in Table 1 show that the current definition of the upper-level problem keeps constant values of the cycles on its minimal bound. This solution proves that the current form of the upper level problem makes fair distribution of the waiting time of the vehicles. Currently, the traffic light cycles y are evaluated to their minimal values of 40 s.

The lower-level problem solutions (x and u) have different values, which vary according to the cycle number (iteration) of the bi-level problem. Decreasing the initial lengths of vehicle queues, the bi-level problem achieves steady state level of queues and they are kept constant, according to cycle number/iteration 3 and 4 regardless that the input flows of vehicles are kept nearly the same. It means that the bi-level problem in a fast way, only for two traffic cycles, works out the disturbances x_0 of waiting vehicles in front of the junctions.

An overall assessment of the benefits of the bi-level problem optimization is made by calculating the amount of vehicles, which are waiting in queues, during the control process. This sum of queues is calculated for each solution of the bi-level problem on the different control iterations. One bi-level solution corresponds to traffic lights control for one cycle. The graphical interpretation of the dynamical increase and decrease of the waiting vehicles for different cycles is presented in Fig. 2 for the illustrated four iterations (four cycles) of solving the bi-level problem. The abscise axis is the number of cycles (calculating iterations), where the index (b) concerns the value of all waiting vehicles in the beginning of the control process. The ordinate axis gives the volume of all vehicles waiting at the queues in the network. The index (e) concerns the optimal values of waiting cars as a solution of the bi-level problem. It is evident that the amount of the initial waiting vehicles decreases considerably.

At the beginning of the control process (before solving the bi-level problem) the sum of all 30 queue lengths is 701 (point 1_b), Fig. 2. After the solution of the bi-level problem (end of first cycle/iteration, point 1_e) this value decreases more than twice to 331. For the beginning of each sequential cycle/iteration the incoming flows x_{in} increase the volume of waiting cars. But after solving the bi-level problem this volume decreases.

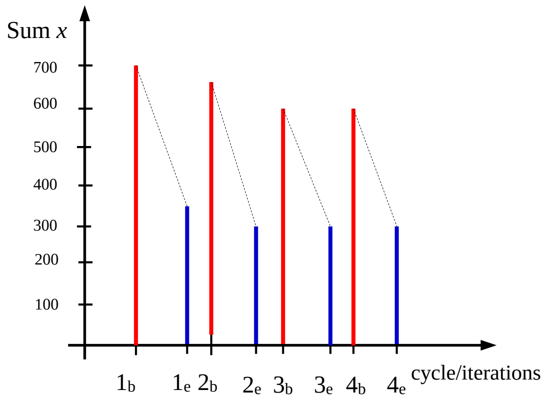

Fig. 2. Sum of vehicle queue lengths

For the case of the second cycle/iteration the initial volume of all waiting vehicles of the 30 traffic flows directions is 672 (point 2_b). After solving the bi-level problem the volume of waiting cars decreases again twice to 315 (point 2_e). The third and fourth cycles/iterations prove that the sequentially usage of the bi-level problem keeps steady state level of vehicles in the urban network, 656 vehicles on the beginning (points 3_b and 4_b) and its decrease to 315 (points 3_e and 4_e).

The dynamics of the sum of all queue lengths and the sum of queues after bi-level optimization are presented in Fig. 3. The upper (red) line is the initial sum of queue lengths. The lower (blue) line is the sum of queue lengths after bi-level optimal solution.

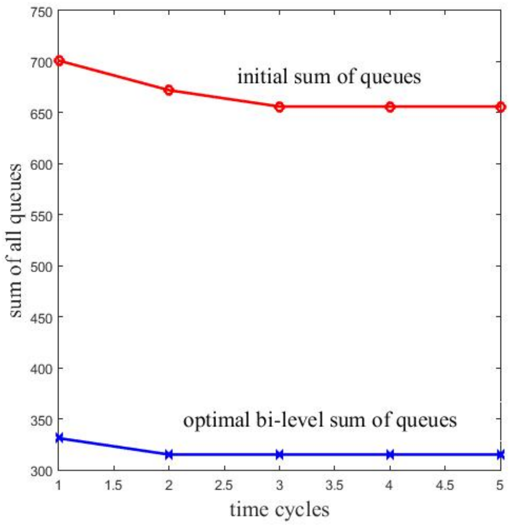

Fig. 3. Dynamics of the sum of all queue lengths per cycle

Fig. 3 shows that the sum of queue lengths decreases more than twice for each cycle, which proves the advantage of the bi-level optimization.

In Figs 4-6 the dynamic changes of some of the queue lengths of vehicles for the junctions of the urban network is illustrated. Fig. 7 represents the green light duration dynamics for a sequence of cycles.

The results in Figs 4-6 illustrate that the bi-level control policy leads to decreasing of all queues in the network, regardless of their positions in the urban junctions. Part of the control impacts (green splits) **u** are illustrated in Fig. 7. The green light splits increase and/or decrease appropriately per cycle, according to the solutions of the bi-level problem.

These simulation results demonstrate abilities for fast control by solving the bi-level problem for each control cycle. The decrease of the vehicle queues is reduced more than twice, which is achieved promptly for the duration of one traffic light cycle.

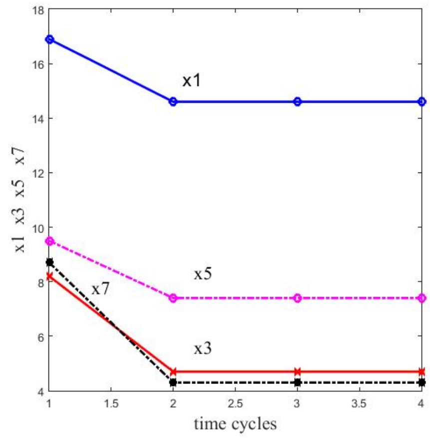

Fig. 4. Queue x1, x3, x5, x7 dynamics

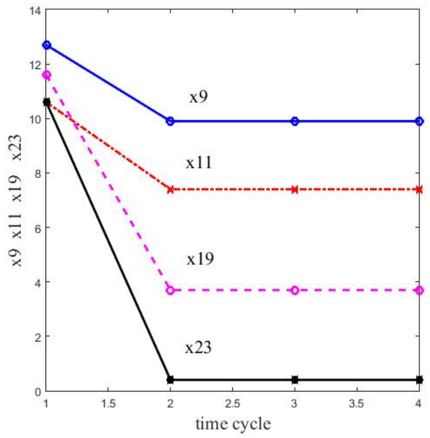

Fig. 5. Queue x9, x11, x19, x23 dynamics

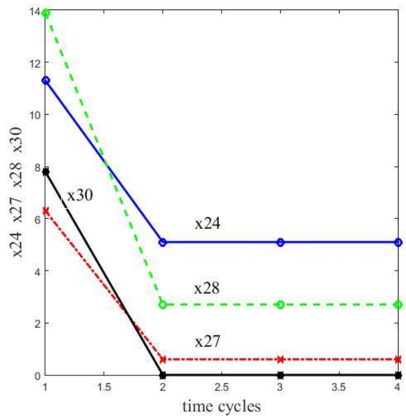

Fig. 6. Queue x24, x27, x28, x30 dynamics

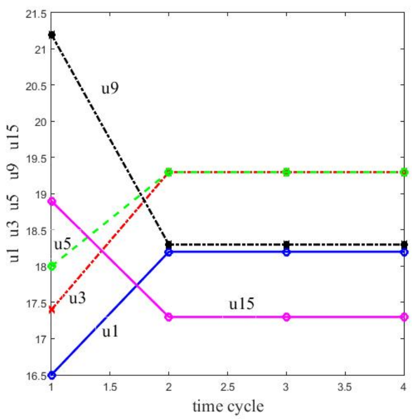

Fig. 7. Green u1, u3, u5, x9, u15 dynamics

6. Conclusion

The added value of this research addresses the implementation of more complex modelling and formalization of traffic control and optimization in urban areas. A bi-level modelling has been applied, which differs from the classical definition of the optimization problem. The application of bi-level modelling allows the control space of the control problem to be extended with both the green splits and the cycles of the traffic lights. Thus, more goals of the traffic control can be satisfied. The current bi-level problem definition allows fair distributing the waiting time between the vehicles in the network. The next added value of the optimization model is the minimization of the queue lengths in front of the junctions that benefits the traffic behavior in the network. Both goal functions cannot be achieved by a classical definition of an optimization problem. The bi-level modelling and optimization has potential for increasing the set of requirements, which the traffic control system has to satisfy. The results from the numerical simulations show decrease of all queues at the junctions in the urban network of the model. These results give evidences that the applied bi-level modelling supports the main research goal for improving the transportation behavior.

The presented results of the numerical simulations concern the network control evaluated for a real case of an urban environment in town of Sofia. The estimated solutions of the traffic dynamics result in sequential improvement in a fast way only by solving and applying twice the bi-level solutions. This is a model's prerequisite for fast implementation of real time traffic control.

A potential extension of this research concerns not only the increase of the scale of the urban network. Additional direction of the development of the bi-level modelling and optimization is the definition and inclusion of the offsets in the upper level problem for increasing the control space. The inclusion of offsets will give rise to opportunity to keep "green wave" control policy for the main streams of the urban transportation. Such potential extensions are related to the complications of the bi-level problems and this will require development of appropriate numerical algorithms for decreasing the computational workload in the problem's solving. This is a mandatory requirement for the real time implementation of the bi-level modelling and control.

***Acknowledgments*:** This work has been supported by project KP06-H37/6 "Modelling and optimization of urban traffic in network of crossroads" with the Bulgarian Research Fund.

References

1. A a v a n i, P., M. K. S a w a n t, S. S a w a n t, R. S. D e s h m u k h. A Review on Adaptive Traffic Controls Systems. – International Journal of Latest Engineering and Management Research (IJLEMR), Vol. **2**, January 2017, Issue 1, pp. 52-57. ISSN: 2455-4847.
2. A b o u d o l a s, K., M. P a p a g e o r g i o u, E. K o s m a t o p o u l o s. Store-and-Forward Based Methods for the Signal Control Problem in Large-Scale Congested Urban Road Networks. – Transportation Research. Part C. Vol. **1**, 2009, pp. 163-174. DOI:10.1016/j.trc.2008.10.002.
3. B a l a b a n o v, A., T. S t o i l o v, Y. B o n e v a. Linear-Quadratic-Gaussian Optimization of Urban Transportation Network with Application to Sofia Traffic Optimization. – Cybernetics and Information Technologies, Vol. **16**, 2016, No 3, pp. 165-184.
4. B e r g, M., A. H e g y i, B. de S c h u t t e r, H. H e l l e n d o o r n. Integrated Traffic Control for Mixed Urban and Free Way Networks: A Model Predictive Control Approach. – EJTIR, Vol. **7**, 2007, No 3, pp. 223-250.
5. B i a n c o, L., M. C a r a m i a, S. G i o r d a n i. A Bilevel Flow Model for Hazmat Transportation Network Design. – Transportation Research Part C: Emerging Technologies, Vol. **17**, April 2009, No 2, pp. 175-196. DOI: 10.1016/j.trc.2008.10.001.
6. B u i l e n k o, V., A. P a k h o m o v a, S. P a k h o m o v. Procedure for Calculating On-Time Duration of the Main Cycle of a Set of Coordinated Traffic Lights. – Transportation Research Procedia, Vol. **20**, 2017, pp. 231-235
7. C a l l e-L a g u n a, A. J., J. D u, H. A. R a k h a. Computing Optimum Traffic Signal Cycle Length Considering Vehicle Delay and Fuel Consumption. – Transportation Research Interdisciplinary Perspectives, Vol. **3**, 2019, 100021.
   http://dx.doi.org/10.1016/j.trip.2019.100021
   https://www.sciencedirect.com/science/article/pii/S2590198219300211?via%3Dihub
8. C h a n g, T.-H., J.-T. L i n. Optimal Signal Timing for an Oversaturated Intersection. – Transp. Res. Part B Methodol., Vol. **34**, 2000, pp. 471-491.

9. C h o w, A. H. F., R. S h a, S. L i. Centralised and Decentralised Signal Timing Optimization Approaches for Network Traffic Control. – Transportation Research Procedia, Vol. **38**, 2019, pp. 222-241. DOI: 10.1016/j.trpro.2019.05.013.
10. C l u n e, A., M. S m i t h, Y. X i a n g. A Theoretical Basis for Implementation of a Quantitative Decision Support System Using Bilevel Optimization. – In: Proc. of 14th International Symposium on Transportation and Traffic Theory, Jerusalem, Israel, 20-23 July, 1999, A. Ceder, Ed., pp. 489-513. ISBN: 0 08 0434487.
11. D a n t a s, L. D., B. F r i e d r i c h. Concurrent Split, Offset and Cycle Control through Model Predictive Control. – In: Proc. of 13th IFAC Symposium on Control in Transportation Systems, the International Federation of Automatic Control, September 12-14, 2012. Sofia, Bulgaria, pp. 24-29. 978-3-902823-13-7, DOI: 10.3182/20120912-3-BG-2031.00006.
12. D i a k a k i, C., M. P a p a g e o r g i o u, K. A b o u d o l a s. A Multivariable Regulator Approach to Traffic-Responsive Network. – Wide Signal Control, Control Engineering Practice, Vol. **10**, 2002, No 2, pp. 183-195.
13. D i a k a k i, C., V. D i n o p o u l o u, K. A b o u d o l a s, M. P a p a g e o r g i o u, E. B e n-S h a b a t, E. S e i d e r, A. L e i b o v. Extensions and New Applications of the Traffic Signal Control Strategy TUC. – Transportation Research Record Journal of the Transportation Research Board. January 2003. DOI: 10.3141/1856-22.
14. D i n o p o u l o u, V., C. D i a k a k i, M. P a p a g e o r g i o u. Applications of the Urban Traffic Control Strategy TUC. – European Journal of Operational Research, Vol. **175**, 2006, pp. 1652-1665. DOI:10.1016/j.ejor.2005.02.032.
15. F a r a h a n i, R. Z., E. M i a n d o a b c h i., W. Y. S z e t o, H. R a s h i d i. Review on Urban Transportation Network Design Problems. – European Journal of Operational Research, Vol. **229**, 1 September 2013, Issue 2, pp. 281-302.
16. G a r c í a-R ó d e n a s, R., M. L. L ó p e z-G a r c í a, M. T. S á n c h e z-R i c o, J., A. L ó p e z-G ó m e z. A Bilevel Approach to Enhance Prefixed Traffic Signal Optimization. – Engineering Applications of Artificial Intelligence, Vol. **84**, 2019, pp. 51-65.
17. G a z i s, D., R. P o t t s. The Oversaturated Intersection. – In: Proc. of 2nd International Symposium on Traffic Theory, London, 1963, pp. 221-237.
18. G e o r g i e v a, P. V., I. P. P o p c h e v, S. N. S t o y a n o v. A Multi-Step Procedure for Asset Allocation in Case of Limited Resources. – Cybernetics and Information Technologies, Vol. **15**, 2015, No 3, pp. 41-50.
19. G o r o d o k i n, V., Z. A l m e t o v a, V. S h e p e l e v. Procedure for Calculating On-Time Duration of the Main Cycle of a Set of Coordinated Traffic Lights. – Transportation Research Procedia, Vol. **20**, 2017, pp. 231-235.
20. H a j b a b a i e, A., R. F. B e n e k o h a l. A Program for Simultaneous Network Signal Timing Optimization and Traffic Assignment. – IEEE Transactions on Intelligent Transportation Systems, Vol. **16**, 2015, pp. 2573-2586.
https://doi.org/10.1109/TITS.2015.2413360
21. H a n, K., Y. S u n, H. L i u, T. L. F r i e s z, T. Y a o. A Bi-Level Model of Dynamic Traffic Signal Control with Continuum Approximation. – Transportation Research Part C: Emerging Technologies, Vol. **55**, 1 June 2015, pp. 409-431. DOI: 10.1016/j.trc.2015.03.037.
22. H a n, K., H. L i u, V. V. G a y a h, T. L. F r i e s z, T. Y a o. A Robust Optimization Approach for Dynamic Traffic Signal Control with Emission Considerations. – Transp. Res. Part C: Emerging Technologies, Vol. **70**, 2016, pp. 3-26.
23. H o o g e n d o o r n, S. P., P. H. L. B o v y. Multiclass Macroscopic Traffic Flow Modelling: A Multilane Generalization Using Gas-Kinetic Theory. – In: Proceedings of the 14th International Symposium on Transportation and Traffic Theory, Jerusalem, Israel, 20-23 July, 1999, A. Ceder Ed., pp. 27-50. ISBN: 0 08 043448 7.
24. I s l a m, S. M. A. B. A., A. H a j b a b a i e. Distributed Coordinated Signal Timing Optimization in Connected Transportation Networks. – Transportation Research Part C: Emerging Technologies, Vol. **80**, 1 July 2017, pp. 272-285. DOI: 10.1016/j.trc.2017.04.017.
25. J a m a l, A., M. T. R a h m a n, H. M. Al-A h m a d i, I. U l l a h, M. Z a h I d. Intelligent Intersection Control for Delay Optimization: Using Meta-Heuristic Search Algorithms. – J. Sustainability, Vol. **12**, 2020, 1896. DOI:10.3390/su12051896.

26. J i a, X., R. H e, C. Z h a n g, H. C h a i. A Bi-Level Programming Model of Liquefied Petroleum Gas Transportation Operation for Urban Road Network by Period-Security. – J. Sustainability, Vol. **10**, 2018, 4714. DOI:10.3390/su10124714.
27. K a m a l, M. A. S., J. I m u r a, A. O h a t a, T. H a y a k a w a. K. A i h a r a. Control of Traffic Signals in a Model Predictive Control Framework. – In: 13th IFAC Symposium on Control in Transportation Systems. The International Federation of Automatic Control, 12-14 September 2012, Sofia, Bulgaria. ISBN: 978-3-902823-13-7/12, DOI:10.3182/20120912-3-BG-2031.00044.
28. K a s h a n i, H. R., G. N. S a r i d i s. Intelligent Control for Urban Traffic Systems. – Automatica, Vol. **19**, 1983, No 2, pp. 191-197.
29. L i, D., Y. S o n g, Q. C h e n. Bilevel Programming for Traffic Signal Coordinated Control Considering Pedestrian Crossing. – Journal of Advanced Transportation, Vol. **2020**, Article ID 3987591. 18 p.
https://doi.org/10.1155/2020/3987591
https://www.hindawi.com/journals/jat/2020/3987591/
30. L i, Z., M. S h a h i d e h p o u r, S. B a h r a m iI r a d, A. K h o d a e i. Optimizing Traffic Signal Settings in Smart Cities. – IEEE Transactions on Smart Grid, Vol. **8**, 2017, pp. 2382-2393.
https://doi.org/10.1109/TSG.2016.2526032
31. L i g h t h i l l, M., G. W h i t h a m. On Kinematic Waves. II. A Theory of Traffic Flow on Long Crowded Roads. – Proceedings of the Royal Society of London. Series A. Mathematical and Physical Sciences, Vol. **229** (1178), 1955, pp. 317-345.
32. M i c h a i l i d i s, I. T., D. M a n o l i s, P. M i c h a i l i d i s, C. D i a k a k i, E. B. K o s m a t o p-o u l o s. A Decentralized Optimization Approach Employing Cooperative Cycle-Regulation in an Intersection-Centric Manner: A Complex Urban Simulative Case Study. – Transportation Research Interdisciplinary Perspectives, Vol. **8**, 2020, 100232.
http://dx.doi.org/10.1016/j.trip.2020.100232
https://www.sciencedirect.com/science/article/pii/S2590198220301433?via%3Dihub
33. M o h a n, R., G. R a m a d u r a i. State-of-the Art of Macroscopic Traffic Flow Modeling. – International Journal of Advances in Engineering Sciences and Applied Mathematics, Vol. **5**, 2013, No 2-3, pp. 158-176. DOI: 10.1007/s12572-013-0087-1.
34. N e s m a c h n o w, S., R. M a s s o b r i o, E. A r r e c h e, C. M u m f o r d, A. C. O l i v e r a, P. J. V i d a l, A. T c h e r n y k h. Traffic Lights Synchronization for Bus Rapid Transit Using a Parallel Evolutionary Algorithm. – Int. J. Transp. Sci. Technol., Vol. **8**, 2019, pp. 53-67.
35. P a p a g e o r g i o u, M. Overview of Road and Motorway Traffic Control Strategies. – IFAC Proceedings Volumes, Vol. **37**, October 2004, Issue 19, pp. 29-40.
https://www.sciencedirect.com/science/article/pii/S1474667017306572
36. P a r k, B. B., N. M. R o u p h a I l, J. S a c k s. Assessment of Stochastic Signal Optimization Method Using Microsimulation. – Trans. Res. Rec., Vol. **1748**. 2001, pp. 40-45.
37. P e n e v a, V., I. P o p c h e v. Models for Weighted Aggregation of Fuzzy Relations to Multicriteria Decision Making Problems. – Cybernetics and Information Technologies, Vol. **6**, 2006, No 3, pp. 3-18.
38. P e n e v a, V., I. P o p c h e v. Multicriteria Decision Making by Fuzzy Relations and Weighting Functions for the Criteria. – Cybernetics and Information Technologies, Vol. **9**, 2009, No 4, pp. 58-71.
39. P o p c h e v, I., I. R a d e v a. A Decision Support Method for Investment Preference Evaluation. – Cybernetics and Information Technologies, Vol. **6**, 2006, No 1, pp. 3-16.
40. Q a d r i, S., M. G o k c e, E. O n e r. State-of-Art Review of Traffic Signal Control Methods: Challenges and Opportunities. – J. European Transport Research Review, 2020.
https://doi.org/10.1186/s12544-020-00439-1
41. R a d i v o j e v i c, M., M. T a n a s k o v i c, Z. S t e v i c. The Adaptive Algorithm of a Four Way Intersection Regulated by Traffic Lights with Four Phases within a Cycle. – Expert Systems with Applications, Vol. **166**, 2021, 114073.
https://doi.org/10.1016/j.eswa.2020.114073
42. R a m e z a n i, M., J. H a d d a d, N. G e r o l iI m i n i s. Macroscopic Traffic Control of a Mixed Urban and Freeway Network. – In: Proc. of 13th IFAC Symposium on Control in Transportation Systems, 12-14 September 2012, Sofia, Bulgaria, pp. 89-94.

43. R i c h a r d s, P. I. Shockwaves on the Highway. – Operations Research, Vol. **4**, 1956, No 1, pp. 42-51.
44. S t o i l o v, T., K. S t o i l o v a. Noniterative Coordination in Multilevel Systems. Dordrecht, Boston, London, Kluwer Academic Publisher, 1999. 268 p. ISBN 0-7923-5879-1.
45. S t o i l o v, T., K. S t o i l o v a, M. P a p a g e o r g i o u, I. P a p a m i c h a i l. Bi-Level Optimization in a Transport Network. – Cybernetics and Information Technologies, Vol. **15**, 2015, No 5, pp. 37-49.
46. S t o i l o v a, K., T. S t o i l o v, K. P a v l o v a. Traffic Management of Urban Network by Bi-Level Optimization. – Journal Information Technologies and Control, 2019, Issue 4, pp. 12-21. Online ISSN: 2367-5357, ISSN: 2367-5357, DOI: 10.7546/itc-2019-0017.
47. S t o i l o v a, K., T. S t o I l o v. Bi-Level Optimizatio Application for Urban Traffic Management. – Annals of Computer Science and Information Systems, Vol. **21**, ISSN 2300-5963. Proc. of 2020 Federated Conference on Computer Science and Information Systems, 6-9 September 2020, Sofia, Bulgaria, pp. 327-336.
http://dx.doi.org/10.15439/978-83-955416-7-4
48. S t o i l o v a, K., T. S t o i l o v. Integrated Management of Transportation by Bi-Level Optimization. – In: Proc. of International Conference Automatics and Informatics (ICAI'20), 1-3 October 2020, Technically Supported by: Technical University of Varna, IEEE by Bulgarian Section and Federation of the Scientific Engineering Unions, Varna, Bulgaria – ICAI2020, pp. 1-6. DOI: 10.1109/ICAI50593.2020.9311360, E-ISBN: 978-1-7281-9308-3.
49. S t o i l o v a, K., T. S t o i l o v, V. I v a n o v. Bi-Level Optimization as a Tool for Implementation of Intelligent Transportation Systems. – Cybernetics and Information Technologies, Vol. **17**, 2017, No 2, pp. 97-105.
50. S u n, D., R. F. B e n e k o h a l. Bi-Level Programming Formulation and Heuristic Solution Approach for Dynamic Traffic Signal Optimization. – J. Computer-Aided Civil and Infrastructure Engineering, Vol. **21**, 2006, pp. 321-333.
51. T a n, M. K., H. S. E. C h u o, R. K. Y. C h i n, K. B. Y e o, K. T. K. T e o. Optimization of Traffic Network Signal Timing Using Decentralized Genetic Algorithm. – In: Proc. of IEEE 2nd International Conference on Automatic Control and Intelligent Systems (I2CACIS'17), Kota Kinabalu, Malaysia, 21-21 October 2017, pp. 62-67.
52. T a n, T., F. B a o, Y. D e n g, A. J i n, Q. D a i, J. W a n g. Cooperative Deep Reinforcement Learning for Large-Scale Traffic Grid Signal Control. – In: IEEE Trans. Cybern., 2019.
53. T a w f i k, C., S. L i m b o u r g. Bilevel Optimization in the Context of Intermodal Pricing: State of Art. – Transportation Research Procedia, Vol. **10**, 2015, pp. 634-643. DOI: 10.1016/j.trpro.2015.09.017.
54. T e t t a m a n t i, T., A. M o h a m m a d i, H. A s a d i, I. V a r g a. A Two-Level Urban Traffic Control for Autonomous Vehicles to Improve Network-Wide Performance. – Transportation Research Procedia, Vol. **27**, 2017, pp. 913-920.
55. V a n K a t w i j k, R. T., B. D e S c h u t t e r, J. H e l l e n d o o r n. Traffic Adaptive Control of a Single Intersection: A Taxonomy of Approaches. – IFAC Proceedings Volumes, Vol. **39**, January 2006, Issue 12, pp. 227-232.
https://doi.org/10.3182/20060829-3-NL-2908.00040
56. W e I, H., G. Z h e n g, V. G a y a h, Z. L i. A Survey on Traffic Signal Control Methods. – ACM, Vol. **1**, 2020, No 1.
https://arxiv.org/abs/1904.08117
57. W u, Y., J. L u, H. C h e n, H. Y a n g. Development of an Optimization Traffic Signal Cycle Length Model for Signalized Intersections in China. – Mathematical Problems in Engineering. Vol. **2015**. Article ID 954295. 9 p.
http://dx.doi.org/10.1155/2015/954295
58. Y i n, F., Y. Z h a o. Optimizing Vehicle Routing via Stackelberg Game Framework and Distributionally Robust Equilibrium Optimization Method. – J. Information Sciences, Vol. **557**, 2021, pp. 84-107.
https://doi.org/10.1016/j.ins.2020.12.057
59. Y i n, H., S. C. W o n g, J. X u, C. K. W o n g. Urban Traffic Flow Prediction Using a Fuzzy Neural Approach. – Transportation Research, Part C, Vol. **10**, 2002, pp. 85-98.

60. Y u, B., L. K o n g, Y. S u n, B. Y a o, Z. G a o. A Bi-Level Programming for Bus Lane Network Design. – Transportation Research Part C: Emerging Technologies, Vol. **55**, 1 June 2015, pp. 310-327. DOI: 10.1016/j.trc.2015.02.014.
61. X i a o-G u a n g, Y., Z. J i n g, W. T a o. Optimal Cycle Calculation Method of Signal Control at Roundabout. – China J. Highw. Transp., Vol. **6**, 2008, pp. 90-95.
62. Y a o, H., X. L i. Decentralized Control of Connected Automated Vehicle Trajectories in Mixed Traffic at an Isolated Signalized Intersection. – Transportation Research Part C, Vol. **121**, 2020, 102846.
https://doi.org/10.1016/j.trc.2020.102846
63. Z h a n g, Y., R. S u. An Optimization Model and Traffic Light Control Scheme for Heterogeneous Traffic Systems. – Transportation Research Part C: Emerging Technologies, Vol. **124**, March 2021, 102911.
64. Z h a o, L., X. P e n g, L. L i, Z. L i. A Fast Signal Timing Algorithm for Individual Oversaturated Intersections. – IEEE Trans. Intell. Transp. Syst., Vol. **12**, 2010, pp. 280-283.
65. **https://yalmip.github.io/command/solvebilevel/**

Received: 25.02.2021; Second Version: 18.05.2021; Accepted: 15.06.2021