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New Ranking Functions for Interval-Valued Intuitionistic Fuzzy Sets and Their Application to Multi-Criteria Decision-Making Problem

Melda Kokoç[1], *Süleyman Ersöz*[2]

[1]*Education and External Relations Coordination Office, Gazi University, 06560, Ankara, Turkey*
[2]*Kırıkkale University, Department of Industrial Engineering, 71450, Kırıkkale, Turkey*
E-mails: meldakokoc@gazi.edu.tr sersoz@kku.edu.tr

***Abstract*:** *Many authors agree that the Interval-Valued Intuitionistic Fuzzy Set (IVIFS) theory generates as realistic as possible evaluation of real-life problems. One of the real-life problems where IVIFSs are often preferred is the Multi-Criteria Decision-Making (MCDM) problem. For this problem, the ranking of values obtained by fuzzing the opinions corresponding to alternatives is an important step, as a failure in ranking may lead to the selection of the wrong alternative. Therefore, the method used for ranking must have high performance. In this article, a new score function S_{KE} and a new accuracy function H_{KE} are developed to overcome the disadvantages of existing ranking functions for IVIFSs. Then, two illustrative examples of MCDM problems are presented to show the application of the proposed functions and to evaluate their effectiveness. Results show that the functions proposed have high performance and they are the eligibility for the MCDM problem.*

***Keywords*:** *Accuracy function, Multi-criteria decision making, interval-valued intuitionistic fuzzy set, score function, ranking.*

1. Introduction

Multi-Criteria Decision-Making (MCDM) is used to select the most suitable one among a number of alternatives or rank the alternatives, by considering different criteria at the same time. In many Decision-Making (DM) processes, judgments must be made within a limited time or in lack of information. Therefore, the judgments expressed by decision-makers are often ambiguous, that is, they contain hesitation or uncertainty. Traditional methods assume that all information about an alternative is expressed with crisp values. But, since most of the decisions are made in an uncertain environment, they should not be expressed with crisp values in real life [1, 2].

Zadeh [5] introduced the Fuzzy Set (FS) theory to cope with the uncertainty. In FS theory, the membership degree is denoted by a value between zero and one [3, 4]. Since FS theory doesn't consider the hesitation of opinion, then

A t a n a s s o v [6] extended the FSs to Intuitionistic Fuzzy Sets (IFSs) by considering hesitation degree. A t a n a s s o v and G a r g o v [7] discovered that degrees expressed by Interval Values (IVs) rather than a single value give more accurate results for real problems. Therefore, they extended IFS to Interval-Valued Intuitionistic Fuzzy Set (IVIFS). In IVIFSs theory, element of the set is represented by three-degree, namely membership, non-membership, and hesitation. In addition, these degrees are symbolized with interval-values.

Many authors have made several studies to develop the IVIFSs theory. X u [8] developed two aggregation operators for fuzzing for IVIFSs. Y o n g b i a o, J u n and X i a o w e n [9] introduced ideal and anti-ideal weighted correlation coefficients, and then they proposed an MCDM method based on these correlation coefficients. In the MCDM method proposed by P a r k et al. [10], an optimization model was created by using the values obtained with the score function to determine the weight of the criteria. Then, alternatives were ranked considering the correlation coefficients between IVIFVs. R a s h i d, F a i z i and Z a f a r [11] introduced a new entropy measure that takes distance into consideration for MCDM problems containing uncertain information. The weights of the alternatives were calculated by the proposed measure, after fuzzing the information about each alternative. W a n et al. [12] developed a model for the MCDM problems with the missing attribute weight. In their work, the weights of each decision-maker were determined considering both the degree of similarity and the degree of proximity. In addition, a linear model was created to objectively obtain the attributes' weights. To produce a consistent preference relationship, L i a o, X u and X i a [13] developed a multiplicative consistency analysis approach. Moreover, an algorithm was presented to reconstruct inconsistent matrices. A new measure of distance was generalized by D ü g e n c i [14]. Later, the TOPSIS method was developed, where the proposed distance measurement was used to calculate the separation measures. In addition, a linear programming model was developed using the relative proximity coefficient to find weights of criteria. In the MCDM model proposed by K o n g et al. [15], the weights of attributes were obtained with a cross-entropy distance and the weights of decision-makers were calculated with a nonlinear optimization model. An artificial bee colony algorithm was developed to solve this nonlinear optimization model. W a n g and W a n [16] introduced an MCDM method using probability and divergence degree. In this method, a linear programming model that operates with interval values was created to specify the weights of decision-makers. After the opinions were fuzzed using the weights of the decision-makers, the probability distribution matrix was produced. Then, collective comprehensive values were obtained by combining the negative and positive distances calculated from this matrix. L i u, Z h e n g and X i o n g [17] expanded entropy and subsethood measures for IVIFSs and discussed the properties required for entropy and subsethood measures. Y e [18] developed the entropy measure to calculate the weights of attributes and proposed a correlation coefficient based on the entropy measure to determine the ideal alternative.

The ranking for IVIFVs is another important research topic in the MCDM problems, and so several studies have been done on the ranking of IVIFVs. X u [8] developed score function $S(.)$ and accuracy function $H(.)$ to rank IVIFVs. Since the score and accuracy functions proposed by [8] didn't rank correctly many IVIFVs, many authors enriched this field by developing the ranking functions that produce more accurate results. Ye [19] presented a method depending on the new accuracy function $M(.)$ to produce a solution for MCDM problems. Nayagam, Muralikrishnan and Sivaraman [20] offered the MCDM method based on the new accuracy function $L(.)$. Bai [21] proposed the score function $I(.)$ and integrated this function to the IVIF-TOPSIS method. Garg [2] introduced a generalized improved score function GIS(.) by including the weighted average of the hesitation degree. Then, an MCDM method based on this score function was presented. Şahin [22] developed an accuracy function $K(.)$ taking the hesitation degree into consideration. Wang and Chen [23] proposed a score function $S_{\mathrm{WC}}(.)$ and improved the MCDM method by integrating both the linear programming and score function. Wang and Chen [24] developed a score function $S_{\mathrm{NWC}}(.)$ and an accuracy function $H_{\mathrm{NWC}}(.)$, and then, they presented an MCDM method utilizing the linear programming approach and the score function together. The aforementioned ranking functions cannot present the correct ranking order for some comparable IVIFVs, and they produce contradictory results. In this paper, a new score function $S_{\mathrm{KE}}(.)$ and a new accuracy function $H_{\mathrm{KE}}(.)$ are proposed to succeed in ranking problems.

The rest of the article is organized as follows. In Section 2, some basic information of the IVIFSs such as definition, relations and operations, aggregation operators, accuracy functions, and score functions are reviewed. In Section 3, the new score function and the new accuracy function are introduced. Moreover, their properties are presented. In Section 4, critical analysis of the existing score functions and accuracy functions are presented. In Section 5, two illustrative examples are presented to demonstrate an application of new functions to MCDM problems. In Section 6, a general evaluation is made and the study is summarized.

2. Preliminaries

2.1. Basic information about IVIFS

Definition 1 [7]. An IvIFS ψ in H is defined by a set where $\lambda = [\underline{\lambda}, \overline{\lambda}]: H \to D[0,1]$ and $\hbar = [\underline{\hbar}, \overline{\hbar}]: H \to D[0,1]$ are named as the membership degree and non-membership degree provided that $0 \le \overline{\lambda} + \overline{\hbar} \le 1$, $\underline{\lambda}, \underline{\hbar} \ge 0$.

Definition 2 [7]. Interval $[1-\overline{\lambda}-\overline{\hbar}, 1-\underline{\lambda}-\underline{\hbar}]$ abbreviated by $[\underline{\pi}, \overline{\pi}]$ is called the hesitation degree.

Definition 3 [7]. For two IVIFVs "$\psi_1 = [\underline{\lambda}_1, \overline{\lambda}_1], [\underline{\hbar}_1, \overline{\hbar}_1]$" and "$\psi_2 = [\underline{\lambda}_2, \overline{\lambda}_2], [\underline{\hbar}_2, \overline{\hbar}_2]$" relations and operations can be defined as below:

(1) $\psi_1 \oplus \psi_2 = \left([\underline{\lambda}_1 + \underline{\lambda}_2 - \underline{\lambda}_1\underline{\lambda}_2, \overline{\lambda}_1 + \overline{\lambda}_2 - \overline{\lambda}_1\overline{\lambda}_2], [\underline{\hbar}_1\underline{\hbar}_2, \overline{\hbar}_1\overline{\hbar}_2]\right)$,

(2) $\psi_1 \otimes \psi_2 = \left(\left[\underline{\lambda}_1\underline{\lambda}_2, \overline{\lambda}_1\overline{\lambda}_2\right], \left[\underline{\hbar}_1 + \underline{\hbar}_2 - \underline{\hbar}_1\underline{\hbar}_2, \overline{\hbar}_1 + \overline{\hbar}_2 - \overline{\hbar}_1\overline{\hbar}_2\right]\right),$

(3) $\alpha\psi = \left(\left[\left(1-(1-\underline{\lambda})^{\alpha}\right), \left(1-(1-\overline{\lambda})^{\alpha}\right)\right], \left[(\underline{\hbar})^{\alpha}, (\overline{\hbar})^{\alpha}\right]\right), \alpha > 0,$

(4) $\psi^{\beta} = \left(\left[(\underline{\lambda})^{\beta}, (\overline{\lambda})^{\beta}\right], \left[\left(1-(1-\underline{\hbar})^{\beta}\right), \left(1-(1-\overline{\hbar})^{\beta}\right)\right]\right), \beta > 0,$

(5) $\psi_1 \subseteq \psi_2 \rightarrow \underline{\lambda}_1 \le \underline{\lambda}_2, \overline{\lambda}_1 \le \overline{\lambda}_2, \underline{\hbar}_1 \ge \underline{\hbar}_2, \overline{\hbar}_1 \ge \overline{\hbar}_2.$

Definition 4 [8]. Some basic aggregation operators for IVIFVs are presented as follows assuming weight vector $\varpi = \left(\varpi_1, \varpi_2, ..., \varpi_n\right)^{\mathrm{T}}$ under condition $\varpi_i \in [0,1]$, $\sum_{i=1}^{n} \varpi_i = 1$.

- IVIF weighted averaging (IVIFWA) operator

(6) $\mathrm{IVIFWA}_{\varpi} = \left(\left[1-\prod_{j=1}^{n}\left(1-\underline{\lambda}_j\right)^{\varpi_j}, 1-\prod_{j=1}^{n}\left(1-\overline{\lambda}_j\right)^{\varpi_j}\right], \left[\prod_{j=1}^{n}\left(\underline{\hbar}_j\right)^{\varpi_j}, \prod_{j=1}^{n}\left(\overline{\hbar}_j\right)^{\varpi_j}\right]\right),$

- IVIF weighted geometric (IVIFWG) operator

(7) $\mathrm{IVIFWG}_{\varpi} = \left(\left[\prod_{j=1}^{n}(\underline{\lambda})^{\varpi_j}, \prod_{j=1}^{n}\left(\overline{\lambda}\right)^{\varpi_j}\right], \left[1-\prod_{j=1}^{n}(1-\underline{\hbar})^{\varpi_j}, 1-\prod_{j=1}^{n}\left(1-\overline{\hbar}\right)^{\varpi_j}\right]\right).$

Theorem 1. [23] Let ψ_1 and ψ_2 be any two IVIFVs, and $F(.)$ is a ranking function of IVIFSs. If $\psi_1 \ne \psi_2$, then $F(\psi_1) \ne F(\psi_2)$.

2.2. Some ranking functions proposed in previous studies

In this section, some accuracy functions and score functions are presented chronologically. Let "$\psi = \left[\underline{\lambda}, \overline{\lambda}\right], \left[\underline{\hbar}, \overline{\hbar}\right]$" be an IVIFV:

- Xu [8] defined a score function presented as in (8),

(8) $$S(\psi) = \frac{\underline{\lambda} - \underline{\hbar} + \overline{\lambda} - \overline{\hbar}}{2};$$

- Xu [8] defined an accuracy function presented as in (9),

(9) $$H(\psi) = \frac{\underline{\lambda} + \overline{\lambda} + \underline{\hbar} + \overline{\hbar}}{2};$$

- Ye [19] defined an accuracy function presented as in (10),

(10) $$M(\psi) = \underline{\lambda} + \overline{\lambda} - 1 + \frac{\underline{\hbar} + \overline{\hbar}}{2};$$

- Nayagam, Muralikrishnan and Sivaraman [20] defined an accuracy function presented as

(11) $$L(\psi) = \frac{\underline{\lambda} + \overline{\lambda} - \overline{\hbar}(1-\overline{\lambda}) - \underline{\hbar}(1-\underline{\lambda})}{2};$$

- B a i [21] defined a score function presented as

(12) $$I(\psi)=\frac{\underline{\lambda}+\underline{\lambda}(1-\underline{\lambda}-\underline{\hbar})+\overline{\lambda}+\overline{\lambda}(1-\overline{\lambda}-\overline{\hbar})}{2};$$

- G a r g [2] defined a generalized score function presented as

(13) $$\mathrm{GIS}(\psi)=\frac{\underline{\lambda}+\overline{\lambda}}{2}+\ell_1\underline{\lambda}(1-\underline{\lambda}-\underline{\hbar})+\ell_2\overline{\lambda}(1-\overline{\lambda}-\overline{\hbar}),\ \text{where}\ \ell_1=1-\ell_2\in[0,1];$$

- Ş a h i n [22] defined an accuracy function presented as

(14) $$K(\psi)=\frac{\underline{\lambda}+\overline{\lambda}(1-\underline{\lambda}-\underline{\hbar})+\overline{\lambda}+\underline{\lambda}(1-\overline{\lambda}-\overline{\hbar})}{2};$$

- W a n g and C h e n [23] defined a score function presented as

(15) $$S_{\mathrm{WC}}(\psi)=\frac{\underline{\lambda}+\overline{\lambda}+\sqrt{\overline{\lambda}\overline{\hbar}}(1-\underline{\lambda}-\underline{\hbar})+\sqrt{\underline{\lambda}c}(1-\overline{\lambda}-\overline{\hbar})}{2};$$

- W a n g and C h e n [24] defined a score function presented as

(16) $$S_{\mathrm{NWC}}(\psi)=\frac{(\underline{\lambda}+\overline{\lambda})(\underline{\lambda}+\underline{\hbar})-(\underline{\hbar}+\overline{\hbar})(\overline{\lambda}+\overline{\hbar})}{2};$$

- W a n g and C h e n [24] defined an accuracy function presented as

(17) $$H_{\mathrm{NWC}}(\psi)=\frac{(1-\underline{\lambda}+\overline{\lambda})(1-\underline{\lambda}-\underline{\hbar})+(1-\underline{\hbar}+\overline{\hbar})(1-\overline{\lambda}-\overline{\hbar})}{2};$$

- J o s h i and K u m a r [25] defined an accuracy function presented as

(18) $$T(\psi)=\frac{\underline{\lambda}(1-\underline{\hbar})+\overline{\lambda}(1-\overline{\hbar})}{2}.$$

3. Introducing the proposed score function and accuracy function

3.1. The new score function

In this section, a new score function $S_{\mathrm{KE}}(.)$ is introduced for ranking IVIFVs and its basic properties are discussed. Assuming "$\psi=\left[\underline{\lambda},\overline{\lambda}\right],\left[\underline{\hbar},\overline{\hbar}\right]$" be an IVIFV, the score function $S_{\mathrm{KE}}(.)$ is presented as

(19) $$S_{\mathrm{KE}}(\psi)=\frac{(\overline{\lambda}-\overline{\hbar})+\left[\left(\underline{\lambda}+\underline{\hbar}\right)\left(\underline{\lambda}-\overline{\hbar}\right)\right]}{2}.$$

Property 1. For any IVIFV "$\psi=\left[\underline{\lambda},\overline{\lambda}\right],\left[\underline{\hbar},\overline{\hbar}\right]$", $S_{\mathrm{KE}}(\psi)\in[-1,1]$.

Property 2. If the IVIFV "$\psi=[1,1],[0,0]$", then $S_{\mathrm{KE}}(\psi)=1$.

P r o o f: Let "$\psi=[1,1],[0,0]$". Based on the new score function we get

(20) $$S_{\mathrm{KE}}(\psi)=\frac{(1-0)+\left[\left(1+0\right)\left(1-0\right)\right]}{2}=1.$$

Property 3. If the IVIFV $\psi = ([0, 0], [1, 1])$, then $S_{\text{KE}}(\psi) = -1.$

Proof: Let $\psi = ([0, 0], [1, 1])$. Based on the new score function we get

(21) $$S_{\text{KE}}(\psi) = \frac{(0-1) + \left[(0+1)(0-1)\right]}{2} = -1.$$

Property 4. If the IVIFV $\psi = ([0.5, 0.5], [0.5, 0.5])$, then $S_{\text{KE}}(\psi) = 0.$

Proof: Let $A = ([0.5, 0.5], [0.5, 0.5])$. Based on the new score function we get

(22) $$S_{\text{KE}}(\psi) = \frac{(0.5-0.5) + \left[(0.5+0.5)(0.5-0.5)\right]}{2} = 0.$$

Property 5. For any IVIFV "$\psi = \left[\underline{\lambda}, \overline{\lambda}\right], \left[\underline{\hbar}, \overline{\hbar}\right]$", $S_{\text{KE}}(\psi) \neq S_{\text{KE}}(\psi^{c})$.

Proof: Let

$\psi = ([0.15, 0.20], [0.65, 0.70])$, $\psi^{\text{c}} = ([0.65, 0.70], [0.15, 0.20])$.

Based on the new score function we get

(23) $$S_{\text{KE}}(\psi) = \frac{(0.20-0.70) + \left[(0.15+0.65)(0.15-0.70)\right]}{2} = -0.47,$$

(24) $$S_{\text{KE}}(\psi^{\text{c}}) = \frac{(0.70-0.20) + \left[(0.65+0.15)(0.65-0.20)\right]}{2} = 0.43.$$

It is proved below that **the new score function presents one value for each IVIFV** by using the Theorem 1.

Proof: Assume that $\psi_1 < \psi_2$, i.e., $\underline{\lambda}_1 < \underline{\lambda}_2$, $\overline{\lambda}_1 < \overline{\lambda}_2$, $\underline{\hbar}_1 > \underline{\hbar}_2$ and $\overline{\hbar}_1 > \overline{\hbar}_2$. Based on the new score function, we get

(25) $$\begin{gathered} S_{\text{KE}}(\psi_1) - S_{\text{KE}}(\psi_2) = \\ = \frac{(\overline{\lambda}_1 - \overline{\hbar}_1) + \left[(\underline{\lambda}_1 + \underline{\hbar}_1)(\underline{\lambda}_1 - \overline{\hbar}_1)\right]}{2} = \\ = \frac{(\overline{\lambda}_2 - \overline{\hbar}_2) + \left[(\underline{\lambda}_2 + \underline{\hbar}_2)(\underline{\lambda}_2 - \overline{\hbar}_2)\right]}{2} = \\ = \frac{\overline{\lambda}_1 - \overline{\hbar}_1 + \underline{\lambda}_1^2 - \underline{\lambda}_1\overline{\hbar}_1 + \underline{\lambda}_1\underline{\hbar}_1 - \overline{\hbar}_1\underline{\hbar}_1 - \overline{\lambda}_2 + \overline{\hbar}_2 - \underline{\lambda}_2^2 + \underline{\lambda}_2\overline{\hbar}_2 - \underline{\lambda}_2\underline{\hbar}_2 + \underline{\hbar}_2\overline{\hbar}_2}{2}. \end{gathered}$$

Because $-\overline{\hbar}_1 < 0$, $-\underline{\lambda}_1\overline{\hbar}_1 < 0$, $-\overline{\hbar}_1\underline{\hbar}_1 < 0$, $-\overline{\lambda}_2 < 0$, $-\underline{\lambda}_2^2 < 0$ and $-\underline{\lambda}_2\underline{\hbar}_2 < 0$ we get the inequality below. Thus, if $\psi_1 < \psi_2$, then $S_{\text{KE}}(\psi_1) \neq S_{\text{KE}}(\psi_2)$. In the same way, if $\psi_1 > \psi_2$, then $S_{\text{KE}}(\psi_1) \neq S_{\text{KE}}(\psi_2)$,

$$S_{KE}(\psi_1) - S_{KE}(\psi_2) < \frac{\overline{\lambda}_1 + \underline{\lambda}_1^2 + \underline{\lambda}_1\underline{\hbar}_1 + \overline{\hbar}_2 + \underline{\lambda}_2\overline{\hbar}_2 + \underline{\hbar}_2\overline{\hbar}_2}{2}.$$

3.2. The new accuracy function

In this section a new accuracy function $H_{KE}(.)$ is introduced for ranking IVIFVs and its basic properties are discussed. Assuming "$\psi = \left[\underline{\lambda}, \overline{\lambda}\right], \left[\underline{\hbar}, \overline{\hbar}\right]$" be an IVIFV, the accuracy function $H_{KE}(.)$ is presented as in (26).

(26) $$H_{KE}(\psi) = \frac{(\underline{\lambda} + \overline{\lambda}) + \alpha(1 + \overline{\lambda} - \overline{\hbar}) + (1-\alpha)\left(1 + \underline{\lambda} - \underline{\hbar}\right)}{4}, \; a \in [0,1].$$

Property 1. For any IVIFV "$\psi = \left[\underline{\lambda}, \overline{\lambda}\right], \left[\underline{\hbar}, \overline{\hbar}\right]$", $H_{KE}(\psi) \in [0,1]$.

Property 2. If the IVIFV "$\psi = [1,1],[0,0]$", then $H_{KE}(\psi) = 1$.

Proof: Let "$\psi = [1,1],[0,0]$". Based on the new accuracy function we get

(27) $$H_{KE}(\psi) = \frac{(1+1) + (\alpha)(1+1-0) + (1-\alpha)\left(1+1-0\right)}{4} = 1.$$

Property 3. If the IVIFV $\psi = \left([0,0],[1,1]\right)$, then $H_{KE}(\psi) = 0$.

Proof: Let $\psi = \left([0,0],[1,1]\right)$. Based on the new accuracy function we get

(28) $$H_{KE}(\psi) = \frac{(0+0) + \alpha(1+0-1) + (1-\alpha)\left(1+0-1\right)}{4} = 0.$$

Property 4. If the IVIFV $\psi = \left([0.5, 0.5],[0.5, 0.5]\right)$, then $H_{KE}(\psi) = 0.5$.

Proof: Let $A = \left([0.5, 0.5],[0.5, 0.5]\right)$. Based on the new accuracy function we get

(29) $$H_{KE}(\psi) = \frac{(0.5+0.5) + \alpha(1+0.5-0.5) + (1-\alpha)\left(1+0.5-0.5\right)}{4} = 0.5.$$

Property 5. For any IVIFV "$\psi = \left[\underline{\lambda}, \overline{\lambda}\right], \left[\underline{\hbar}, \overline{\hbar}\right]$", $H_{KE}(\psi) \neq H_{KE}(\psi^c)$.

Proof: Let

$$\psi = \left([0.15, 0.20],[0.65, 0.70]\right), \; \psi^c = \left([0.65, 0.70],[0.15, 0.20]\right).$$

Based on the new score function we get

(30) $$H_{KE}(\psi) = \frac{(0.15+0.20) + \alpha(1+0.20-0.70) + (1-\alpha)\left(1+0.15-0.65\right)}{4} = 0.21,$$

(31) $$H_{KE}(\psi^c) = \frac{(0.65+0.70) + \alpha(1+0.70-0.20) + (1-\alpha)\left(1+0.65-0.15\right)}{4} = 0.71.$$

It is proved below that **the new accuracy function presents one value for each IVIFV** by using the Theorem 1.

Proof: Assume that $\psi_1 < \psi_2$, i.e., $\underline{\lambda}_1 < \underline{\lambda}_2$, $\overline{\lambda}_1 < \overline{\lambda}_2$, $\underline{\hbar}_1 > \underline{\hbar}_2$ and $\overline{\hbar}_1 > \overline{\hbar}_2$. Based on the new accuracy function where $\alpha \in [0,1]$, we get

$$\begin{aligned}
&H_{\mathrm{KE}}(\psi_1) - H_{\mathrm{KE}}(\psi_2) = \\
&= \frac{(\underline{\lambda}_1 + \overline{\lambda}_1) + \alpha(1 + \overline{\lambda}_1 - \overline{\hbar}_1) + (1-\alpha)\left(1 + \underline{\lambda}_1 - \underline{\hbar}_1\right)}{4} = \\
&= \frac{(\underline{\lambda}_2 + \overline{\lambda}_2) + \alpha(1 + \overline{\lambda}_2 - \overline{\hbar}_2) + (1-\alpha)\left(1 + \underline{\lambda}_2 - \underline{\hbar}_2\right)}{4} = \\
&= \frac{\underline{\lambda}_1 + \overline{\lambda}_1 + \alpha + \alpha\overline{\lambda}_1 - \alpha\overline{\hbar}_1 + 1 + \underline{\lambda}_1 - \underline{\hbar}_1 - \alpha - \alpha\underline{\lambda}_1 + \alpha\underline{\hbar}_1}{4} + \\
&+ \frac{-\underline{\lambda}_2 - \overline{\lambda}_2 - \alpha - \alpha\overline{\lambda}_2 + \alpha\overline{\hbar}_2 - 1 - \underline{\lambda}_2 + \underline{\hbar}_2 + \alpha + \alpha\underline{\lambda}_2 - \alpha\underline{\hbar}_2}{4} = \\
&= \frac{(2-\alpha)\underline{\lambda}_1 + (1+\alpha)\overline{\lambda}_1 - \alpha(\overline{\hbar}_1 - \overline{\hbar}_2) - (2-\alpha)\underline{\lambda}_2 - (1+\alpha)\overline{\lambda}_2 + (1-\alpha)(\underline{\hbar}_2 - \underline{\hbar}_1)}{4},
\end{aligned} \tag{32}$$

because $-\alpha(\overline{\hbar}_1 - \overline{\hbar}_2) < 0$, $-(2-\alpha)\underline{\lambda}_2 < 0$, $-(1+\alpha)\overline{\lambda}_2 < 0$ and $(1-\alpha)(\underline{\hbar}_2 - \underline{\hbar}_1) < 0$ we get the inequality below. Thus, if $\psi_1 < \psi_2$, then $H_{\mathrm{KE}}(\psi_1) \neq H_{\mathrm{KE}}(\psi_2)$. In the same way, if $\psi_1 > \psi_2$, then $H_{\mathrm{KE}}(\psi_1) \neq H_{\mathrm{KE}}(\psi_2)$,

$$H_{\mathrm{KE}}(\psi_1) - H_{\mathrm{KE}}(\psi_2) < \frac{(2-\alpha)\underline{\lambda}_1 + (1+\alpha)\overline{\lambda}_1}{4}.$$

4. Critical analysis of the existing score functions and accuracy functions of IVIFSs

In this section, some examples are given to show that existing score functions proposed by [2, 8, 21-23] and accuracy functions proposed by [8, 19, 20, 22, 24, 25] can't present correct ranking in some cases. Assuming ψ_1 and ψ_2 be two IVIFVs, ψ_1 and ψ_2 used for comparison in the examples are presented in Table 1.

Table 1. IVIFN pairs used in comparison examples

Example No	ψ_1	ψ_2
Example 1	([0.10, 0.20], [0.30, 0.50])	([0.10, 0.20], [0.20, 0.30])
Example 2	([0.15, 0.20], [0.65, 0.70])	([0.65, 0.70], [0.15, 0.20])
Example 3	([0.00, 0.40], [0.30, 0.40])	([0.00, 0.30], [0.30, 0.40])
Example 4	([0.00, 0.00], [0.10, 0.30])	([0.00, 0.00], [0.20, 0.40])
Example 5	([0.20, 0.60], [0.20, 0.40])	([0.30, 0.50], [0.10, 0.50])

Example 1. Clearly $\psi_1 \subset \psi_2$.By applying $H(.)$ defined in [8] we get $H(\psi_1) = 0.55$ and $H(\psi_2) = 0.40$. Therefore $H(\psi_1) > H(\psi_2)$ which is the opposite. By applying $M(.)$ defined in [19] we get $M(\psi_1) = -0.30$ and $M(\psi_2) = -0.45$. Hence $M(\psi_1) > M(\psi_2)$ which is the opposite.

Example 2. Clearly $\psi_1 \subset \psi_2$. By applying $H(.)$ defined in [8] we get $H(\psi_1) = 0.85$ and $H(\psi_2) = 0.85$. By applying $H_{\text{NWC}}(.)$ defined in [24] we get $H_{\text{NWC}}(\psi_1) = 0.158$ and $H_{\text{NWC}}(\psi_2) = 0.158$. Since $H(\psi_1) = H(\psi_2)$ and $H_{\text{NWC}}(\psi_1) = H_{\text{NWC}}(\psi_2)$, it is not known which alternative is better.

Example 3. Clearly $\psi_1 \supset \psi_2$. By applying $I(.)$ defined in [21], $K(.)$ defined in [22], $S_{\text{WC}}(.)$ defined in [23] and $T(.)$ defined in [25] we get $I(\psi_1) = I(\psi_2) = 0$, $K(\psi_1) = K(\psi_2) = 0$, $S_{\text{WC}}(\psi_1) = S_{\text{WC}}(\psi_2) = 0$ and $T(\psi_1) = T(\psi_2) = 0$. In this case, ranking or selection cannot be made because both values are equal to each other.

Example 4. Clearly $\psi_1 \supset \psi_2$. By applying $\text{GIS}(.)$ defined in [2] we get $\text{GIS}(\psi_1) = \text{GIS}(\psi_2) = 0.30$. Similarly, it is not known which alternative is better. By applying $S_{\text{NWC}}(.)$ defined in [24] we get $S_{\text{NWC}}(\psi_1) = -0.22$ and $S_{\text{NWC}}(\psi_2) = -0.20$. The score function S_{NWC} fails to rank these two alternatives.

Example 5. Clearly $\psi_1 \supset \psi_2$. By applying $S(.)$ defined in [8], $L(.)$ defined in [20] and $T(.)$ defined in [25], we get $S(\psi_1) = S(\psi_2) = 0.10$, $L(\psi_1) = L(\psi_2) = 0.24$, $T(\psi_1) = T(\psi_2) = 0.26$, respectively. It cannot be determined which alternative is better.

The examples are applied to all of the ranking functions compared in addition to new proposed functions. Results of new score function and new accuracy function are explained below. In addition, the performances of all functions for five examples are summarized in Fig. 1. It is seen that the new score function and the new accuracy function have the highest performance and they overcome the drawback of ranking functions proposed in [2, 8, 19-25] to distinguish the IVIFVs.

By applying new score function and new accuracy function to Example 1, we get $S_{\text{KE}}(\psi_1) = -0.23$, $S_{\text{KE}}(\psi_2) = -0.08$ and $H_{\text{KE}}(\psi_1) = 0.26$, $H_{\text{KE}}(\psi_2) = 0.30$ for $\alpha = 0.6$. Thus, ψ_2 is better than ψ_1 for both functions.

By applying new score function and new accuracy function to Example 2, we get $S_{\text{KE}}(\psi_1) = -0.47$, $S_{\text{KE}}(\psi_2) = 0.43$ and $H_{\text{KE}}(\psi_1) = 0.213$, $H_{\text{KE}}(\psi_2) = 0.713$ for $\alpha = 0.6$. Thus, ψ_2 is better than ψ_1 for both functions.

By applying new score function and new accuracy function to Example 3, we get $S_{\text{KE}}(\psi_1) = -0.06$, $S_{\text{KE}}(\psi_2) = -0.11$ and $H_{\text{KE}}(\psi_1) = 0.32$, $H_{\text{KE}}(\psi_2) = 0.28$ for $\alpha = 0.6$. Thus, ψ_1 is better than ψ_2 for both functions.

By applying new score function and new accuracy function to Example 4, we get $S_{\text{KE}}(\psi_1) = -0.165$, $S_{\text{KE}}(\psi_2) = -0.24$ and $H_{\text{KE}}(\psi_1) = 0.195$, $H_{\text{KE}}(\psi_2) = 0.17$ for $\alpha = 0.6$. Thus, ψ_1 is better than ψ_2 for both functions.

By applying new score function and new accuracy function to Example 5, we get $S_{\mathrm{KE}}(\psi_1)=0.06$, $S_{\mathrm{KE}}(\psi_2)=-0.04$ and $H_{\mathrm{KE}}(\psi_1)=0.48$, $H_{\mathrm{KE}}(\psi_2)=0.47$ for $\alpha=0.6$. Thus, ψ_1 is better than ψ_2 for both functions.

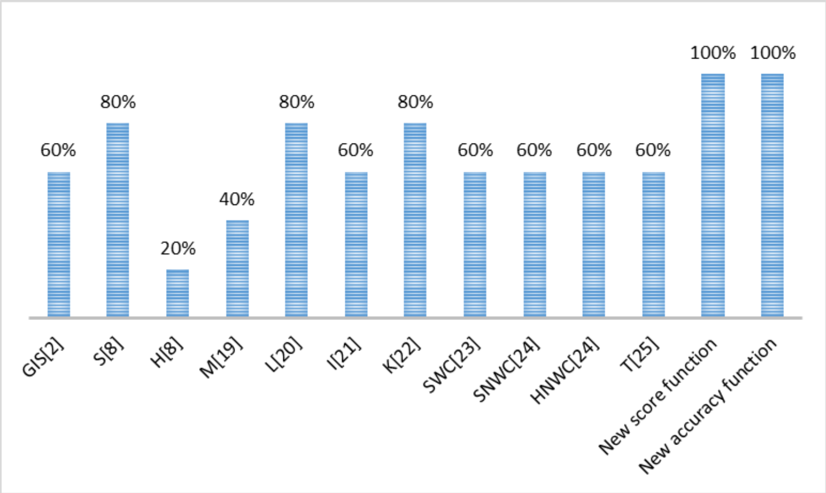

Fig. 1. Comparative analysis result for different ranking functions

5. Application of proposed functions to MCDM problems

In this section, the application of the proposed score function and accuracy function to the multi-criteria decision-making problem is shown. For this, an approach is offered for the MCDM problem, in which the weights of the criteria are known. In order to measure the effectiveness of the proposed functions and to make comparisons, two illustrative examples used in previous studies are selected.

Assuming that $\psi_i=\{\psi_1,\psi_2,\ldots,\psi_m\}$ be set of the alternatives, $\xi_j=\{\xi_1,\xi_2,\ldots,\xi_n\}$ be set of the criteria, and $\varpi=\{\varpi_1,\varpi_2,\ldots,\varpi_n\}$ be set of the criteria' weights where $\varpi\in[0,1]$ and $\sum_{j=1}^{n}\varpi j=1$, the procedure for the proposed MCDM approach is summarized in below:

Step 1. Aggregate the judgments corresponding to each alternative using the weights of criteria, i.e., obtain the average value.

Step 2. Calculate the score value or accuracy value using the score function or accuracy function.

Step 3. Rank the alternatives using score values or accuracy values.

5.1. Illustrative Example 1

An example adapted from [26] and discussed in [2, 17, 18, 20, 23] is selected as a first example to explain the application of the proposed functions to MCDM. Suppose that the alternative ψ_i, $i=1,2,3,4,$ the criteria ξ_j, $j=1,2,3,$ and the weights of criteria are 0.35, 0.25, and 0.40. The decision-maker evaluates these four possible alternatives by using the IVIFV under the three criteria. Then the pairwise matrix is obtained as seen in Table 2, using the judgements of decision-maker.

Table 2. Decision matrix $D_{4\times 3}(x_{ij})$

ψ_i	ξ_1	ξ_2	ξ_3
ψ_1	[0.4, 0.5], [0.3, 0.4]	[0.4, 0.6], [0.2, 0.4]	[0.1, 0.3], [0.5, 0.6]
ψ_2	[0.6, 0.7], [0.2, 0.3]	[0.6, 0.7], [0.2, 0.3]	[0.4, 0.8], [0.1, 0.2]
ψ_3	[0.3, 0.6], [0.3, 0.4]	[0.5, 0.6], [0.3, 0.4]	[0.4, 0.5], [0.1, 0.3]
ψ_4	[0.7, 0.8], [0.1, 0.2]	[0.6, 0.7], [0.1, 0.3]	[0.3, 0.4], [0.1, 0.2]

The weighted arithmetic average value $\tilde{\gamma}_i$ for ψ_i is obtained using IVIFWA operator (6). For example, the arithmetic average value is computed for alternative ψ_1 as follows:

$$\tilde{\gamma}_1 = \left(\begin{array}{l} \left[\left[1-\left((1-0.4)^{0.35}(1-0.4)^{0.25}(1-0.1)^{0.40}\right)\right],\left[1-\left((1-0.5)^{0.35}(1-0.6)^{0.25}(1-0.3)^{0.40}\right)\right]\right], \\ \left[\left[\left((0.3)^{0.35}(0.2)^{0.25}(0.5)^{0.40}\right)\right],\left[\left((0.4)^{0.35}(0.4)^{0.25}(0.6)^{0.40}\right)\right]\right] \end{array} \right) =$$

$$= \left[0.294, 0.459\right], \left[0.333, 0.470\right],$$

$$\tilde{\gamma}_1 = \left[0.294, 0.459\right], \left[0.333, 0.470\right],$$

$$\tilde{\gamma}_2 = \left[0.530, 0.745\right], \left[0.152, 0.255\right],$$

$$\tilde{\gamma}_3 = \left[0.395, 0.563\right], \left[0.193, 0.357\right],$$

$$\tilde{\gamma}_4 = \left[0.548, 0.657\right], \left[0.100, 0.221\right].$$

Then, score values $S_{\mathrm{KE}}(\tilde{\gamma}_i)$ and accuracy values $H_{\mathrm{KE}}(\tilde{\gamma}_i)$ of the weighted arithmetic average values are calculated by using (19) and (26), respectively, as in the Table 3.

Table 3. Score and accuracy values for $\tilde{\gamma}_i$

$\tilde{\gamma}_i$	$S_{\mathrm{KE}}(\tilde{\gamma}_i)$	$H_{\mathrm{KE}}(\tilde{\gamma}_i)_{\alpha=0.5}$	$H_{\mathrm{KE}}(\tilde{\gamma}_i)_{\alpha=0.3}$	$H_{\mathrm{KE}}(\tilde{\gamma}_i)_{\alpha=0.7}$
$\tilde{\gamma}_1$	–0.061	0.432	0.431	0.433
$\tilde{\gamma}_2$	0.338	0.677	0.672	0.683
$\tilde{\gamma}_3$	0.114	0.540	0.540	0.541
$\tilde{\gamma}_4$	0.323	0.661	0.662	0.661

Now, the ranking of the alternative is reviewed with weighted geometric average value. The weighted geometric average value $\hat{\gamma}_i$ for ψ_i, $i=1,2,3,4$ is obtained using the IVIFWG (7). For example, weighted geometric average value is computed for alternative ψ_1 as follows:

$$\hat{\gamma}_1 = \left(\begin{array}{l} \left[\left[\left((0.4)^{0.35}(0.4)^{0.25}(0.1)^{0.40}\right)\right], \left[\left((0.5)^{0.35}(0.6)^{0.25}(0.3)^{0.40}\right)\right]\right], \\ \left[\left[1-\left((1-0.3)^{0.35}(1-0.2)^{0.25}(1-0.5)^{0.40}\right)\right], \left[1-\left((1-0.4)^{0.35}(1-0.4)^{0.25}(1-0.6)^{0.40}\right)\right]\right] \end{array} \right) =$$

$$= \left[0.230, 0.427\right], \left[0.367, 0.490\right],$$

$$\hat{\gamma}_1 = [0.230, 0.427], [0.367, 0.490],$$

$$\hat{\gamma}_2 = [0.510, 0.738], [0.161, 0.262],$$

$$\hat{\gamma}_3 = [0.382, 0.558], [0.226, 0.362],$$

$$\hat{\gamma}_4 = [0.480, 0.586], [0.100, 0.226].$$

Then, score values $S_{\text{KE}}(\hat{\gamma}_i)$ and accuracy values $H_{\text{KE}}(\hat{\gamma}_i)$ of the weighted geometric average values are calculated by using (19) and (26), respectively, as in the Table 4.

Table 4. Score and accuracy values for $\hat{\gamma}_i$

$\hat{\gamma}_i$	$S_{\text{KE}}(\hat{\gamma}_i)$	$H_{\text{KE}}(\hat{\gamma}_i)_{\alpha=0.5}$	$H_{\text{KE}}(\hat{\gamma}_i)_{\alpha=0.3}$	$H_{\text{KE}}(\hat{\gamma}_i)_{\alpha=0.7}$
$\hat{\gamma}_1$	–0.109	0.389	0.385	0.393
$\hat{\gamma}_2$	0.322	0.665	0.659	0.672
$\hat{\gamma}_3$	0.104	0.529	0.527	0.531
$\hat{\gamma}_4$	0.254	0.609	0.610	0.608

When the results are analysed, it is realized that the same ranking is obtained by using both the IVIFWA and IVIFWG operators as $\psi_2 > \psi_4 > \psi_3 > \psi_1$. Moreover, the accuracy function is calculated with three alpha (α) ratios which are pessimistic ($\alpha = 0.3$), optimistic ($\alpha = 0.7$), and neutral ($\alpha = 0.5$), and it is seen that accuracy function offers the same ranking as the score function for these three ratios. Finally, it is identified that ranking obtained by proposed functions coincide with the ones shown in [2, 19, 20, 22, 25].

5.2. Illustrative Example 2

The example discussed in [21] is selected as the second example to present the application of the proposed functions to MCDM problem. Suppose that the alternative ψ_i, $i = 1, 2, 3, 4, 5,$ the criteria ξ_j, $j = 1, 2, 3, 4, 5, 6,$ and the weights of criteria are 0.20, 0.10, 0.25, 0.10, 0.15, 0.20, respectively. Decision-maker evaluates five possible alternatives according to six criteria and then the decision matrix is formed by using the IVIFVs values as in Table 5.

Table 5. Decision matrix $D_{5\times 6}(x_{ij})$

ψ_i	ξ_1	ξ_2	ξ_3
ψ_1	[0.2, 0.3], [0.4, 0.5]	[0.6, 0.7], [0.2, 0.3]	[0.4, 0.5], [0.2, 0.4]
ψ_2	[0.6, 0.7], [0.2, 0.3]	[0.5, 0.6], [0.1, 0.3]	[0.6, 0.7], [0.2, 0.3]
ψ_3	[0.4, 0.5], [0.3, 0.4]	[0.7, 0.8], [0.1, 0.2]	[0.5, 0.6], [0.3, 0.4]
ψ_4	[0.6, 0.7], [0.2, 0.3]	[0.5, 0.7], [0.1, 0.3]	[0.7, 0.8], [0.1, 0.2]
ψ_5	[0.5, 0.6], [0.3, 0.5]	[0.3, 0.4], [0.3, 0.5]	[0.6, 0.7], [0.1, 0.3]
ψ_i	ξ_4	ξ_5	ξ_6
ψ_1	[0.7, 0.8], [0.1, 0.2]	[0.1, 0.3], [0.5, 0.6]	[0.5, 0.7], [0.2, 0.3]
ψ_2	[0.6, 0.7], [0.1, 0.2]	[0.3, 0.4], [0.5, 0.6]	[0.4, 0.7], [0.1, 0.2]
ψ_3	[0.6, 0.7], [0.1, 0.3]	[0.4, 0.5], [0.3, 0.4]	[0.3, 0.5], [0.1, 0.3]
ψ_4	[0.3, 0.4], [0.1, 0.2]	[0.5, 0.6], [0.1, 0.3]	[0.7, 0.8], [0.1, 0.2]
ψ_5	[0.6, 0.8], [0.1, 0.2]	[0.6, 0.7], [0.2, 0.3]	[0.5, 0.6], [0.2, 0.4]

The weighted arithmetic average value $\tilde{\gamma}_i$ for ψ_i is obtained using IVIFWA operator (7) as follows. Then, score values $S_{\text{KE}}(\tilde{\gamma}_i)$ and accuracy values $H_{\text{KE}}(\tilde{\gamma}_i)$ of the weighted arithmetic average values are calculated by using (19) and (26), respectively, as in the Table 6.

$$\tilde{\gamma}_1 = [0.417, 0.560], [0.246, 0.380],$$
$$\tilde{\gamma}_2 = [0.518, 0.657], [0.174, 0.295],$$
$$\tilde{\gamma}_3 = [0.470, 0.590], [0.193, 0.342],$$
$$\tilde{\gamma}_4 = [0.607, 0.720], [0.115, 0.240],$$
$$\tilde{\gamma}_5 = [0.537, 0.654], [0.177, 0.356].$$

Table 6. Score and accuracy values for $\tilde{\gamma}_i$

$\tilde{\gamma}_i$	$S_{\text{KE}}(\tilde{\gamma}_i)$	$H_{\text{KE}}(\tilde{\gamma}_i)_{\alpha=0.5}$	$H_{\text{KE}}(\tilde{\gamma}_i)_{\alpha=0.3}$	$H_{\text{KE}}(\tilde{\gamma}_i)_{\alpha=0.7}$
$\tilde{\gamma}_1$	0.102	0.538	0.537	0.538
$\tilde{\gamma}_2$	0.258	0.632	0.31	0.633
$\tilde{\gamma}_3$	0.166	0.581	0.582	0.579
$\tilde{\gamma}_4$	0.373	0.703	0.704	0.703
$\tilde{\gamma}_5$	0.214	0.630	0.633	0.627

When the results of the second illustrative example are analyzed using Table 6, it is seen that the ranking presented as $\psi_4 > \psi_2 > \psi_5 > \psi_3 > \psi_1$ by the score function or accuracy function with a neutral and optimistic alpha ratio is consistent with [21]. The accuracy function with pessimistic α ratio ranked alternatives as $\psi_4 > \psi_5 > \psi_2 > \psi_3 > \psi_1$.

The effects of different alpha ratios on the ranking are analyzed and their corresponding accuracy values are summarized in Table 7. From Table 7, it is deduced that with the increase of α from 0 to 1 the accuracy values of Criterion 1 and Criterion 2 increase whereas accuracy values of Criterion 3, Criterion 4, and Criterion 5 decrease. Moreover, when the rankings are examined, it is seen that the rank changes as $\psi_4 > \psi_2 > \psi_5 > \psi_3 > \psi_1$, when alpha ratio is less than 0.4, $\psi_4 > \psi_2 = \psi_5 > \psi_3 > \psi_1$, when alpha ratio is equal to 0.4, and $\psi_4 > \psi_5 > \psi_2 > \psi_3 > \psi_1$, when alpha ratio is greater than 0.4. Similarly, ψ_4 is first in all three rankings. As can be seen from the results, the proposed accuracy function will help the decision maker to evaluate alternatives based on the situation of the preference environment.

Table 7. Accuracy values that change according to α

α	0.00	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.00
$H_{KE}(\tilde{\gamma}_1)$	0.537	0.537	0.537	0.537	0.538	0.538	0.538	0.538	0.538	0.539	0.539
$H_{KE}(\tilde{\gamma}_2)$	0.630	0.630	0.631	0.631	**0.632**	0.632	0.633	0.633	0.633	0.634	0.634
$H_{KE}(\tilde{\gamma}_3)$	0.584	0.584	0.583	0.582	0.581	0.581	0.580	0.579	0.578	0.578	0.577
$H_{KE}(\tilde{\gamma}_4)$	0.705	0.705	0.704	0.704	0.704	0.703	0.703	0.703	0.702	0.702	0.702
$H_{KE}(\tilde{\gamma}_5)$	0.638	0.636	0.635	0.633	**0.632**	0.630	0.628	0.627	0.625	0.624	0.622

6. Conclusion

In this study, a new score function and a new accuracy function are developed to correctly rank IVIFVs. Some of the ranking functions suggested in previous studies are compared with the proposed new functions through various examples. When the results of the examples are examined, it is seen that the ranking functions suggested in the previous studies do not achieve the correct result for several examples whereas the score function and accuracy function proposed in this study are successful in all ranking examples.

Moreover, two illustrative examples are solved by proposed functions to demonstrate that the proposed functions can be used suitably for MCDM problems. To compare the effectiveness of the proposed functions, the examples are selected from previous studies. In the application process, a weighted aggregation operator is used to aggregate information about alternatives. Then, score values or accuracy values are calculated using the proposed score function or accuracy function from aggregated information. In light of these values, best suitable alternative can be selected or alternatives can be ranked. The inferences obtained from the results and the outstanding contributions of this study are listed below.

- A new score function and a new accuracy function overcoming the deficiencies of the existing ranking functions are developed for IVIFVs.
- New functions can be utilized not only for ranking IVIFVs but also by integrating them into different models.
- MCDM approach developed based on new functions gives the same results as the other methods in previous studies. This shows that the simple approach based on ranking function with high performance present the same ranking with the models required a lot of processing, such as TOPSIS.

- New functions offer consistent results for real-life problems.

Consequently, our proposed functions overcome the deficiency of the existing ranking functions and present the correct ranking order for all comparable IVIFVs. Because of the ranking problem of IVIFVs is a significant topic in several fields, the new score function and accuracy function will be very beneficial and will have many applications in all areas.

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