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Robust Gain-Scheduled PID Control: A Parameter Dependent BMI Solution

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***Abstract*: *In control practices, problems of parametric or time-varying uncertainties must be dealt with. Robust control based on norm theory and convex and non-convex optimization algorithms is a powerful tool to solve these problems in theory, but it is employed rarely in applications. In most engineering cases, Proportional-Integration-Derivative (PID) control is still the most popular method for its easy-to-tune and controllable properties. The control method proposed in this paper integrates the PID control into robust control formulation as a robust Structured Static Output Feedback (SSOF) problem of Linear-Parameter-Varying (LPV) systems, which can be converted into a Parameter Dependent Bilinear-Matrix-Inequality (PDBMI) optimization problem. A convex-concave decomposition based method is given to solve the proposed PDBMI problem. The proposed solution has a simple structure in PID form and can guarantee stability and robustness of the system being controlled in the whole operation range with less conservativeness than existing solution.*

***Keywords*: *Robust control, gain scheduling, Proportional-Integration-Derivative (PID) control, Bilinear-Matrix-Inequality (BMI), Linear-Matrix-Inequality (LMI), Linear-Parameter-Varying (LPV) system.*

1. Introduction

With the development of control theory and engineering, more advanced control technologies are invented by the control community. However when it comes to control applications in the reality, most of the plants are still controlled by Proportional-Integration-Derivative (PID) based control methods [1]. The class of PID controllers has some advantages beyond most of the advanced control methods: first, it is easy to tune because it has only three parameters in every single channel and each of them has a clear physical meaning; second, knowledge about controlled plant can be applied conveniently to determine the control structure, i.e. which outputs are feedback to the controller. However, the PID control method is only valid for Linear-Time-Invariant (LTI) system with Single-Input-Single-Output (SISO) in theory. While in control practice, most of the controlled plants are nonlinear, time-variant and uncertain to some degree. For these cases, the PID control with constant gains would not work well, and then an "engineering-oriented" method called "Gain-Scheduled (GS) PID" is introduced to solve the problem [2]. The traditional GS PID control uses a "divide and conquer" philosophy, i.e., the nonlinear time-varying plant is linearized at several frozen points and thus converted into a family of LTI plants. Then a bank of PID controllers is designed for the LTI family. After that, an interpolation or switching of the PID gains produces a gain-scheduling [3]. The GS PID has been proven to be effective in many engineering cases. However, when it comes to theoretical aspects, GS PID is not valid, because there is no guarantee on robustness performance and even stability for the whole operation range [4].

To overcome the disadvantages above, Shamma proposed the LPV system in his doctoral thesis [5]. LPV system has the same form as LTI system but the system matrices are varying with some parameters while the nonlinearity is reflected by such variation. Therefore it can be used to control nonlinear time-variant systems and the linear control theory can be extended to LPV system [6]. The authors of this paper have recently developed a modeling and control method for a bio-inspired morphing fixed-wing unmanned aerial vehicle based on the LPV system [7]. The most fruitful aspect should belong to the robust control of LPV system proposed by Wu et al. [8] and Becker and Packard [9]. However, most of research works mainly focus on the Dynamic Output Feedback (DOF) control or State Feedback (SF) control problem which can be solved via LMI solution. Similar to the LTI case, the DOF and SF are well-established in theory but often difficult to be implemented in applications. This is because DOF often leads to dynamic controllers of high order and plant states are not always available for SF.

From an applicable viewpoint, combining the theoretical foundation of linear robust control and the intuitive and practicability of GS PID control, robust GS PID control would be a satisfactory solution. Several literatures [10-14] on the problem can be found, but most of the researches are based on the Quadratic Stability (QS) theory, which uses a single Lyapunov function along with a controller with fixed gains [10] solved via Bilinear-Matrix-Inequality (BMI) for the whole operation envelope, which would be much conservative. [12, 14] propose robust GS PID control for Affine LPV (ALPV) systems with polytope uncertainties. However, the affine assumption cannot be held in some cases. Similar to Static Output Feedback (SOF) control of LTI system, the main difficulty is to solve the BMIs, especially for parameter-dependent BMIs in the case of robust GS PID control.

This paper proposes a LPV based design method for robust GS PID control, which is based on the parameter-dependent Lyapunov theory, and furthermore, formulates the problem as a GS SSOF control of LPV system. Then a convex-concave decomposition based algorithm is proposed to solve the PDBMIs. The numerical result shows that the proposed solution is feasible and less conservative than the solution in [10]. The main works in this paper are based on the research results in literature [10, 11]. We extend them to the parameter dependent case with a structured controller, which is less conservative and more practical.

2. Preliminaries

2.1. LPV System and Its robust stability

The plant considered throughout this paper can be described by LPV system as follows:

(1)
$$\begin{cases} \dot{x} = A(\theta)x + B_1(\theta)w + B_2(\theta)u, \\ z = C_1(\theta)x + D_{11}(\theta)w + D_{12}(\theta)u, \\ y = C_2(\theta)x + D_{21}(\theta)w, \end{cases}$$

where $x \in R^{n_x}$ denotes the state vector, $w \in R^{n_w}$ and $u \in R^{n_u}$ represent the disturbance and control input vectors, respectively, $y \in R^{n_y}$ and $z \in R^{n_z}$ are the measured and controlled output vectors, respectively.

$\theta \in \Theta \subset R^{n_\theta}$ stands for the vector of varying parameters. The parameters and their variation rates are both bounded, i.e., Θ is a hyper rectangle and $\beta = \dot{\theta} \in \Omega \subset R^{n_\theta}$, with Ω being a hyper rectangle as well.

LPV system is indeed a sub-kind of nonlinear system because of parameter-variation [6]. The robust stability problem of LPV system is often dealt with using the Quadratic Stability (QS) theory, which provides a single Lyapunov function for the whole parameter trajectory, and thus leads to a conservative result. The method used in this paper is based on a Parameter-Dependent QS (PDQS) theory, which can be described below.

Lemma 1. The LPV system as

(2)
$$\dot{x} = A(\theta)x,$$

is parameter dependent quadratically stable if and only if there exists a parameter dependent symmetric matrix *P* meeting the following conditions, $\theta \in \Theta$:

(3)
$$A^{\mathrm{T}}(\theta)P(\theta) + P(\theta)A(\theta) + \dot{P} < 0,$$
$$P(\theta) > 0, \quad \theta \in \Theta.$$

Proof: This can be proved by choosing a quadratic Lyapunov function as follows:

(4)
$$V(x) = x^{\mathrm{T}}P(\theta)x.$$

And then using the Lyapunov stability theory, i.e., letting the following conditions hold:

(5)
$$V(x(t)) > 0,$$
$$\frac{dV(x(t))}{dt} < 0.$$

It will be easy to prove that (5) is equivalent to (3).

Based on Lemma 1, the bound real lemma (BRL) for LPV system can be obtained.

Theorem 1. The LPV system as

$$\begin{bmatrix} \dot{x} \\ y \end{bmatrix} = \begin{bmatrix} A(\theta) & B(\theta) \\ C(\theta) & D(\theta) \end{bmatrix} \begin{bmatrix} x \\ u \end{bmatrix} \tag{6}$$

is parameter dependent quadratic stable with H_∞ norm less than γ, if and only if there exists a parameter dependent symmetric matrix P satisfying the following LMIs:

$$\begin{bmatrix} A^{\mathrm{T}}P + PA + \dot{P} & PB & C^{\mathrm{T}} \\ B^{\mathrm{T}}P & -\gamma I & D^{\mathrm{T}} \\ C & D & -\gamma I \end{bmatrix} < 0, \tag{7}$$

$$P > 0, \ \ \theta \in \Theta.$$

Here all of the matrix variables are functions of θ, and as so the symbol of function is omitted for the sake of convenience.

Theorem 1 can be proven using Lemma 1 and the BRL condition for LTI system, the proof can be found in [15].

2.2. Problem statement

The problem considered in this paper is to design a practical PID or PI controller for system (1) to make sure that the closed system satisfies the conditions in Theorem 1, which can be summarized as Problem 1.

Problem 1. For system (1), denoted as $G(\theta)$, find a structured PID (or PI) controller denoted as $K(\theta)$ that can be described as follows:

$$\begin{cases} \dot{x}_I = y, \\ u = K_{\mathrm{I}}S_{\mathrm{I}}x_{\mathrm{I}} + K_{\mathrm{P}}S_{\mathrm{P}}y + K_{\mathrm{D}}S_{\mathrm{D}}\dot{y}. \end{cases} \tag{8}$$

Try to make sure that the closed loop system

$$T_{\mathrm{cl}} = \mathrm{LFT}(G, K), \tag{9}$$

meets the conditions in Theorem 1.

In the Problem 1, K_{P}, K_{I} and K_{D} are parameter dependent matrix gains in the Proportional, Integral and Derivative channel, respectively. S_{P}, S_{I} and S_{D} are diagonal matrices with the diagonal elements being either 0 or 1. These matrices will act as the structured matrices for the corresponding channel, which determine which outputs feedback to the controller. For example, a structured integral matrix as

$$S_{\mathrm{I}} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

means the 1st and 3rd outputs are feedback to controller in the integral channel. Moreover, for instance, forcing the matrix K_{D} to null will lead to a PI controller.

3. Robust controller design

3.1. Robust SOF control

Similar to the LTI case, a PID control design problem can always be converted to a Static Output Feedback (SOF) control design problem. Thus the SOF control problem is investigated before solving the PID control problem. The following theorem can be derived from Theorem 1 by extending the result in [10] to the parameter dependent case.

Theorem 2. The system (1) can be stabilized via parameter dependent SOF with an H_∞ norm of the closed loop system less than γ if and only if there exist a parameter dependent matrix K_y and parameter dependent positive definite symmetric matrix P such that the following conditions are satisfied:

$$\begin{bmatrix} A_{cl}^{T}P + PA_{cl} + \dot{P} & PB_{cl} & C_{cl}^{T} \\ B_{cl}^{T}P & -\gamma I & D_{cl}^{T} \\ C_{cl} & D_{cl} & -\gamma I \end{bmatrix} < 0, \tag{10}$$

where

$$A_{cl} = A + B_2K_yC_2,\ B_{cl} = B_1 + B_2K_yD_{21},$$
$$C_{cl} = C_1 + D_{12}K_yC_2,\ D_{cl} = D_{11},$$

and the varying parameter θ is omitted for brevity.

Proof: Let $u=K_y*y$ and substitute it into (1). Then the closed loop system one can get is as follows:

$$\begin{cases} \dot{x} = A_{cl}(\theta)x + B_{cl}(\theta)w, \\ z = C_{cl}(\theta)x + D_{cl}(\theta)w, \end{cases}$$

in which,

$$A_{cl} = A + B_2K_yC_2,\ B_{cl} = B_1 + B_2K_yD_{21},$$
$$C_{cl} = C_1 + D_{12}K_yC_2,\ D_{cl} = D_{11}.$$

So, according to Theorem 1, Theorem 2 is proved.

3.2. Robust GS PID control

Based on Theorem 2, the solution of Problem 1 in PID case can be derived into the following theorem.

Theorem 3. The PID case of Problem 1 is solvable if there exist a parameter dependent positive definite symmetry matrix P and a parameter dependent matrix Ky such that the following conditions are met.

$$\begin{bmatrix} \tilde{A}_{cl}^{T}P + P\tilde{A}_{cl} + \dot{P} & P\tilde{B}_{cl} & \tilde{C}_{cl}^{T} \\ \tilde{B}_{cl}^{T}P & -\gamma I & \tilde{D}_{cl}^{T} \\ \tilde{C}_{cl} & \tilde{D}_{cl} & -\gamma I \end{bmatrix} < 0, \tag{11}$$

where

$$\tilde{A}_{\text{cl}} = \tilde{A} + \tilde{B}_2 K_y \tilde{C}_2,\ \tilde{B}_{\text{cl}} = \tilde{B}_1 + \tilde{B}_2 K_y \tilde{D}_{21},$$
$$\tilde{C}_{\text{cl}} = \tilde{C}_1 + D_{12} K_y \tilde{C}_2,\ D_{\text{cl}} = D_{11},$$

with

$$\tilde{A} = \begin{bmatrix} A & 0 \\ C_2 & 0 \end{bmatrix},\ \tilde{B}_1 = \begin{bmatrix} B_1 \\ 0 \end{bmatrix},\ \tilde{B}_2 = \begin{bmatrix} B_2 \\ 0 \end{bmatrix},$$

(12) $$\tilde{C}_1 = \begin{bmatrix} C_1 & 0 \end{bmatrix},\ \tilde{C}_2 = \begin{bmatrix} S_{\text{P}} C_2 & 0 \\ 0 & S_{\text{I}} \\ S_{\text{D}}(C_2 A + \dot{C}_2) & 0 \end{bmatrix},$$

$$\tilde{D}_{21} = \begin{bmatrix} 0 \\ 0 \\ S_{\text{D}} C_2 B_1 \end{bmatrix},$$

$$K_y = \begin{bmatrix} K_1 \\ K_2 \\ K_3 \end{bmatrix},\ K_i \in R^{n_u \times n_y},\ i = 1, 2, 3,$$

and $I + S_{\text{D}} C_2 B_2 K_3$ is invertible.

If Problem 1 is solvable, a possible set of PID gains can be expressed as follows:

(13) $$\begin{cases} K_{\text{D}} = K_3 / (I + S_{\text{D}} C_2 B_2 K_3), \\ K_{\text{P}} = (I - K_{\text{D}} S_{\text{D}} C_2 B_2) K_1, \\ K_{\text{I}} = (I - K_{\text{D}} S_{\text{D}} C_2 B_2) K_2. \end{cases}$$

Proof: Inserting system (1) into (8), the following equations can be derived:

(14) $$\dot{x}_{\text{I}} = C_2 x,$$

(15) $$u = K_{\text{P}} S_{\text{P}} C_2 x + K_{\text{I}} S_{\text{I}} x_I + K_{\text{D}} S_{\text{D}} C_2 (Ax + B_1 w + B_2 u) = \\ = \left(I - K_{\text{D}} S_{\text{D}} C_2 B_2\right)^{-1} (K_{\text{I}} S_{\text{I}} x_{\text{I}} + (K_{\text{P}} S_{\text{P}} C_2 + K_{\text{D}} S_{\text{D}} \dot{C}_2 + \\ + K_{\text{D}} S_{\text{D}} C_2 A) x + K_{\text{D}} S_{\text{D}} C_2 B_1 w).$$

Let $\eta = \begin{bmatrix} x & x_I \end{bmatrix}^{\text{T}}$, define the system matrices as (12) and formulate a new system as

(16) $$\tilde{S} = \begin{cases} \dot{\eta} = \tilde{A}\eta + \tilde{B}_1 w + \tilde{B}_2 u, \\ z = \tilde{C}_1 \eta + D_{11} w + D_{12} u, \\ \tilde{y} = \tilde{C}_2 \eta + \tilde{D}_{21} w. \end{cases}$$

Let $\Delta = \left(I - K_{\text{D}} S_{\text{D}} C_2 B_2\right)^{-1}$, then K_y can be rewritten as

$$K_y = \Delta \begin{bmatrix} K_\mathrm{P} \\ K_\mathrm{I} \\ K_\mathrm{D} \end{bmatrix}. \tag{17}$$

Then (15) can be written as

$$u = K_y\left(\tilde{C}_2\eta + \tilde{D}_{21}w\right) = K_y\tilde{y}. \tag{18}$$

Thus Problem 1 is equivalent to the parameter dependent SOF problem of system (16). The PID gain matrices can be solved through (17) or (13).

Theorem 3 can be used to solve the PID case of Problem 1 in theory, but when it comes to engineering, Theorem 3 may not be applicable for the following reasons:

1. Derivative of the output matrix C_2 is indispensable, but explicit relationship between C_2 and the varying parameters is often unavailable.

2. Realization of the derivative action often leads to high frequency dynamics (poles with large amplitude), which is difficult to be implemented in restricted hardware.

3. $I+S_\mathrm{D}C_2B_2K_3$ must be invertible in the whole parameter region, which cannot be guaranteed in the design progress.

3.3. Robust GS PI Control

PI control is usually much more applicable in applications and the following Theorem can be used to solve the PI case of Problem 1.

Theorem 4. The PI case of Problem 1 is solvable if there exist a parameter dependent positive definite symmetry matrix P and a parameter dependent matrix K_y such that the following conditions are satisfied.

$$\begin{bmatrix} \tilde{A}_\mathrm{cl}^\mathrm{T}P + P\tilde{A}_\mathrm{cl} + \dot{P} & P\tilde{B}_\mathrm{cl} & \tilde{C}_\mathrm{cl}^\mathrm{T} \\ \tilde{B}_\mathrm{cl}^\mathrm{T}P & -\gamma I & \tilde{D}_\mathrm{cl}^\mathrm{T} \\ \tilde{C}_\mathrm{cl} & \tilde{D}_\mathrm{cl} & -\gamma I \end{bmatrix} < 0, \tag{19}$$

where

$$\tilde{A}_\mathrm{cl} = \tilde{A} + \tilde{B}_2K_y\tilde{C}_2,\ \tilde{B}_\mathrm{cl} = \tilde{B}_1 + \tilde{B}_2K_yD_{21},$$
$$\tilde{C}_\mathrm{cl} = \tilde{C}_1 + D_{12}K_y\tilde{C}_2,\ D_\mathrm{cl} = D_{11},$$

with

$$\tilde{A} = \begin{bmatrix} A & 0 \\ C_2 & 0 \end{bmatrix}, \tilde{B}_1 = \begin{bmatrix} B_1 \\ 0 \end{bmatrix}, \tilde{B}_2 = \begin{bmatrix} B_2 \\ 0 \end{bmatrix},$$
$$\tilde{C}_1 = \begin{bmatrix} C_1 & 0 \end{bmatrix}, \tilde{C}_2 = \begin{bmatrix} S_\mathrm{P}C_2 & 0 \\ 0 & S_\mathrm{I} \end{bmatrix}, \tag{20}$$
$$K_y = \begin{bmatrix} K_1 \\ K_2 \end{bmatrix},\ K_i \in R^{n_u \times n_y},\ i = 1, 2.$$

Then the solution of Problem 1 in PI case is

$$(21)\qquad \begin{cases} K_{\mathrm{P}} = K_1, \\ K_{\mathrm{I}} = K_2. \end{cases}$$

Theorem 4 can be readily proven by forcing K_{D} to zero along the track in Theorem 3.

It is noticed that, the proposed solution in Theorem 3 and Theorem 4 converts the robust GS PID control problem into a parameter dependent SOF problem, which will lead to convex optimization problem with parameter dependent BMI constraints. The following section in this paper focuses on solving the parameter dependent BMIs.

4. The proposed PDBMI solution

4.1. Quadratic decomposition of BMI

This section presents a solution of BMI for a kind of optimization problem with BMI constraints as follows:

$$(22)\qquad \begin{cases} \min\ f(x), \\ \text{s.t. } (X^{\mathrm{T}}Y + Y^{\mathrm{T}}X) \le 0, \\ x \in \Omega, \end{cases}$$

where $X: R^m \to R^{n\times n}$ is a matrix function of x as well as Y and f is a convex function of x.

The BMI constraint in (22) is a frequent case arisen from robust control theory, including the SOF problem. The following results can convert such BMI constraint into an LMI constraint which can be solved efficiently by means of interior-point methods using some open-source software tools such as SeDuMi [16].

Lemma 2. The BMI constraints in (22) can be decomposed into the difference of two quadratic matrix functions as follows:

$$(23)\qquad X^{\mathrm{T}}Y + Y^{\mathrm{T}}X = \frac{1}{2}\left[Q(X+Y) - Q(X-Y)\right],$$

where

$$(24)\qquad Q(F) = F^{\mathrm{T}}F,$$

is a quadratic function of matrix F.

Proof:

$$\frac{1}{2}\left[Q(X+Y) - Q(X-Y)\right] =$$

$$= \frac{1}{2}\left[(X+Y)^{\mathrm{T}}(X+Y) - (X-Y)^{\mathrm{T}}(X-Y)\right] =$$

$$= \frac{1}{2}\left[(X^{\mathrm{T}}X + X^{\mathrm{T}}Y + Y^{\mathrm{T}}X + Y^{\mathrm{T}}Y) - (X^{\mathrm{T}}X - X^{\mathrm{T}}Y - Y^{\mathrm{T}}X + Y^{\mathrm{T}}Y)\right] =$$

$$= \frac{1}{2}\left[2 * X^{\mathrm{T}}Y + 2 * Y^{\mathrm{T}}X\right] = X^{\mathrm{T}}Y + Y^{\mathrm{T}}X,$$

i.e., the right side equals the left side, Lemma 2 is proved.

Theorem 5. For matrix value mapping:

$$X, Y : R^m \to R^{n\times n},$$

The following inequality holds:

(25) $$X^{\mathrm{T}}Y + XY^{\mathrm{T}} \le Q(X) + Q(Y),$$

and "=" holds only when

(26) $$\det(X - Y) = 0.$$

Proof: Let

(27) $$F = X - Y.$$

Using the property of quadratic matrix, we can get the following condition,

(28) $$Q(F) = F^{\mathrm{T}}F \ge 0,$$

in which the "=" holds only when det(F)=0,

$$Q(F) \ge 0 \Leftrightarrow (X - Y)^{\mathrm{T}}(X - Y) \ge 0 \Leftrightarrow$$
$$\Leftrightarrow X^{\mathrm{T}}X + Y^{\mathrm{T}}Y - X^{\mathrm{T}}Y - Y^{\mathrm{T}}X \ge 0 \Leftrightarrow$$
$$\Leftrightarrow X^{\mathrm{T}}Y + Y^{\mathrm{T}}X \le X^{\mathrm{T}}X + Y^{\mathrm{T}}Y \Leftrightarrow (25).$$

Finally, Theorem 5 is proven.

Then the optimization problem of (22) can be converted to convex problem with LMI constraints in the following steps.

Using Lemma 1, the inequality in (22) can be reformulated as

(29) $$Q(X + Y) - Q(X - Y) \le 0.$$

Suppose there is a feasible solution of X_0 and Y_0, let

(30) $$M = X + Y,\ N = X - Y,\ N_0 = X_0 - Y_0,$$

(31) $$G = Q(M),\ H = Q(N),$$

(32) $$H_0 = N^{\mathrm{T}}N_0 + N_0^{\mathrm{T}}N - N_0^{\mathrm{T}}N_0.$$

Using Theorem 5, we can get

(33) $$H_0 \le H.$$

Therefore

(34) $$G - H_0 \le 0,$$

is a sufficient condition of the BMI constraint in (22). The equation (34) can be written as:

(35) $$M^{\mathrm{T}}M - H_0 \le 0.$$

Using the Schur Complement Theorem [15], (35) is equivalent to the following LMI:

(36) $$\begin{bmatrix} H_0 & M^{\mathrm{T}} \\ M & I \end{bmatrix} \ge 0.$$

Then, the problem of (22) can be converted to a convex optimization problem with LMI constraint.

4.2. Application on the SOF problem

In this section, we will apply the quadratic decomposition method to the parameter dependent H_∞ SOF problem in Theorem 2, the following algorithm can be derived.

Step 1. Find an initial feasible solution

It is often difficult to find a feasible solution for (10) straightforward. The following steps can be performed to determine an initial solution.

Step 1.1. Find a feasible solution for the following LMI:

(37)
$$X_0 A^{\mathrm{T}} + AX_0 + F_0^{\mathrm{T}} B_2^{\mathrm{T}} + B_2 F_0 < 0,$$
$$X_0 > 0,$$

in which F_0 is parameter dependent but X_0 is constant. Actually, a feasible state feedback gain F_0 is solved with a fixed Lyapunov function.

Step 1.2. Let

(38)
$$K_0 = F_0 / X_0 * C_2^+,$$

where C_2^+ is the Moore-Penrose inverse of C_2. Then use K_0 as K_y to minimize γ with the constraint of inequality (10), if any solution of P_0^* and γ_0^* is obtained, then (K_0, P_0^*, γ_0^*) is a feasible solution for inequality (10).

Step 2. Apply the Quadratic Decomposition iteratively to solve the BMI

Using procedures in Subsection 4.1 with the feasible solution solved in Step 1 as an initial point, the BMI in (10) can be converted to the following LMI:

(39)
$$\begin{bmatrix} H_k - \dot{P} & -PB_1 & -C_{\mathrm{F}}^{\mathrm{T}} & (A_{\mathrm{F}} + P)^{\mathrm{T}} \\ -B_1^{\mathrm{T}} P & \gamma I & -D_{11}^{\mathrm{T}} & 0 \\ -C_{\mathrm{F}} & -D_{11} & -\gamma I & 0 \\ A_{\mathrm{F}} + P & 0 & 0 & 2I \end{bmatrix} \geq 0,$$

where

(40)
$$H_k = \frac{[(A_{\mathrm{F}} - P)^{\mathrm{T}} (A_{\mathrm{F}k} - P_k) + (A_{\mathrm{F}k} - P_k)^{\mathrm{T}} (A_{\mathrm{F}} - P) - (A_{\mathrm{F}k} - P_k)^{\mathrm{T}} (A_{\mathrm{F}k} - P_k)]}{2},$$

and

(41)
$$A_{\mathrm{F}} = A + B_2 K_y C_2,$$

(42)
$$C_{\mathrm{F}} = A + D_{12} K_y C_2,$$

in which variables with subscript k stand for the expressions calculated using the results in the k-th iteration.

Then, let $k = k+1$ and denote the solution of (39) with subscript k to continue the iteration of Step 2 until the stop condition of

(43)
$$|\gamma_{k+1} - \gamma_k| < \xi,$$

is achieved, where ξ is a small value near 0 (will be chosen empirically as 0.0005 in this paper).

5. Numerical example

The LPV system in the example of [10] is used to illustrate the solution proposed in this paper so as to generate comparisons to the solution proposed in [10]. Assuming that

(44) $$K_y = K_{y0} + p_1 K_{y1} + p_2 K_{y2},$$

and

(45) $$P = P_0 + p_1 P_1 + p_2 P_2,$$

are consistent to the problem in [10], the structured matrices are chosen as follows:

(46) $$S_{\mathrm{I}} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \; S_{\mathrm{P}} = I.$$

Using the solution based on QS in [10] with a fixed gain matrix K_y, an optimal value of

(47) $$\gamma=0.6843$$

is achieved. By virtue of the proposed solution, a parameter dependent gain matrix is obtained using (44):

(48) $$\begin{cases} K_{y_0} = \begin{bmatrix} -4.6, -6.4, 2.71, 4.7, -1.63 \\ -7.16, -5.18, 2.48, -1.06, -2.16 \end{bmatrix}, \\ K_{y_1} = \begin{bmatrix} -67.8, -54.88, 25, 5.43, -14.8 \\ 31.8, 24.8, -11.4, -1.13, 7.7485 \end{bmatrix}, \\ K_{y_2} = \begin{bmatrix} -8.05, -4.74, 4.51, -4, -6.65 \\ -11.3, -7.8, -2.86, -0.01, -2.77 \end{bmatrix}, \end{cases}$$

and an optimal value of

(49) $$\gamma=0.6245$$

is obtained. It is obvious that the obtained value is less than the result presented in the solution in [10] as shown in (47).

6. Conclusions

In this paper, the robust GS PID control problem for LPV system is investigated and a solution based on parameter dependent Lyapunov function is proposed, which leads to a convex optimization problem with parameter dependent BMI constraints. A Quadratic Decomposition based algorithm is proposed and applied to the robust GS SOF problem. The numerical example shows that the proposed solution is effective and less conservative than the solution in [10].

However it is also noticed that, Theorem 5 leads to a sufficient condition to transform BMIs to LMIs. That is to say, the algorithm is also conservative and the result in this paper may not be the optimal solution but a suboptimal one, instead.

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